The problem of foundations of measurement

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Abstract - Given the common assumption that measurement plays an important role in the foundation of science, the paper analyzes the possibility that Measurement Science, and therefore measurement itself, can be properly founded. The realist and the representational positions are analyzed at this regards: the conclusion, that such positions unavoidably lead to paradoxical situations, opens the discussion for a new epistemology of measurement, whose characteristics and interpretation are sketched here but are still largely matter of investigation.

Keywords: Measurement Science; foundations of measurement, uncertainty in measurement

1. The problem of foundation

The traditional image of science is related to the metaphor of a *building*, progressively erected with the contribution of the organized collection of the results obtained by researchers. Such a metaphor is not conceptually neutral: rather, its choice reveals the assumption of the hypothesis that the "building of science" is inherently *provided with foundations*. The term "epistemology" itself, usually deemed as synonymous of Philosophy of Knowledge, and more specifically of Philosophy of Science, etymologically originates from the same metaphor of *standing* (the Latin *sistere*, the Greek *histanai*) *on* (the Greek *epi*). The epistemological and rhetoric strategy of axiomatization, as originated from the Euclidean geometry, is paradigmatic: the building blocks of a theory, i.e., its theorems, are inferentially obtained from the chosen axioms, which thus play the role of foundational elements for the theory itself. This foundational function is characterized in terms of not only consistency (foundations must allow to erect the correct building) and completeness (foundations must allow to erect the whole building), but also minimality (foundations must include only what is required to erect the building), therefore recognizing the importance of *clearly distinguishing foundations from development*.

The heritage of formal sciences deeply influenced also *empirical* sciences: on one hand, the belief arose that the axiomatization of the body of knowledge related to a scientific discipline is to be considered the ultimate step for the development of that discipline (as in the case of Newtonian mechanics, axiomatized by Lagrange and Hamilton), thus providing a strong argument to support the conception of Physics as the methodological paradigm for all the sciences; on the other hand, and more generally, the quest for foundations kept to pervade the philosophical instances accompanying the development of empirical sciences. Again, the terminological habits are revealing, as the idiomatic expressions "foundations of *x*", "fundamentals of *x*", "basics of *x*", … are usually taken as synonyms of discourses on primitive concepts, from which it should be possible to derive all the (theoretical and practical) applications of *x*. All the empirical sciences should replicate the development process of Physics, providing themselves with foundations and unifying thus their methodologies. The neo-positivistic school, whose influence has been determinant for the Philosophy of Science during the XX century, defined the most explicit programme in this direction: its manifest [1], published in 1929 with the title "The scientific conception of the world", states that the neo-positivistic conception is aimed at the methodological unification of science by means of the common denominator of the possibility of foundations (indeed, the main chapter of the manifest is organized in the following paragraphs: 1. The foundations of Arithmetic, 2. The foundations of Physics, 3.The foundations of Geometry, 4. The foundations of Biology and Psychology, 5. The foundations of Social Sciences).

The (usually implicit) hypothesis that such "fundamentals" are definitive truths on which any further scientific development can be cumulated has been traditionally integral part of the physical sciences, in particular after the extraordinary results obtained by the Newtonian mechanics during the XVIII and XIX centuries. As late as 1899 A.A.Michelson expressed this position by stating that "the more important fundamental laws and facts of physical science have all been discovered, and these are so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote (…) Our future discoveries must be looked for in the sixth place of decimals" (to consider how firmly such "fundamental laws and facts" were established, remember that in the first years of the XX century the radically new ideas of quantization and relativity were proposed…).

The dialectic between "normal science" and "revolutions" identified by T. Kuhn [2] is plausibly still compatible with this standpoint, and makes it only relative to paradigms: the walls of a building can begin cracking; those who have worked their whole life to erect the building hardly will recognize as a better option to tear it down and build a brand new one on different foundations. On the other hand, if a community emerges aimed at re-founding the building, generally it will not be able to share its objectives and planning strategies with the previously established one.

As a consequence of this *faith in the cumulative progress* of science, it is not amazing that an important part of Philosophy of Science has been devoted to the search of the "fundamentals" and the discussion on their ability to play the role of "good foundations" for empirical sciences. In particular, the Hume's problem on the justification of inductive reasoning can be interpreted as an investigation of the relation between "facts" and "laws", as expressed by singular and universal assertions respectively, in terms of the ability of the former *to constitute a basis* for the latter. This standpoint pervaded so many philosophical conceptions, as the traditional distinctions of primary vs. secondary qualities and observational vs. theoretical terms witness, that led the neo-positivistic school to take the sensation as the only basis for any scientific construction and even refusing "scientific meaning" to any form of knowledge not founded in this way.

2. Some different positions on the possibility of foundations in measurement

In its deemed role of objective and empirical means for acquiring information, measurement received a peculiar function of *protocol of truth* to give solid foundations to the scientific knowledge (consider the importance traditionally recognized to the so-called "crucial experiments", whose interpretation has been usually considered depending on the precision of the available measurement results, as the previously quoted concept of discoveries to be looked for "in the sixth place of decimals" assumes): to reach truth is the aim of knowledge and measurement is the operative means to get true data. On the other hand, precisely the concept of *truth in measurement* is a discriminating one, and perhaps *the* discriminating one, among different positions that can be assumed on the possibility of foundations in measurement.

It is a recognized fact that science seeks simplicity, customarily adopted as preference, and sometimes even as a confirmation, criterion. Therefore it is not surprising that also from a terminological point of view most scientists privilege simplicity, and are used to speak about, for example, "quantitative phenomena" or "linear systems". On the other hand, the short form of these common idiomatic expressions masks a disputation in the scientific community, which can be outlined in the distinct interpretations that such expressions admit.

- A first position assumes that "quantitative phenomenon" must be understood literally: the particular phenomenon under analysis has the inherent characteristic of being quantitative.

- The second position recognizes instead that this idiom is just an elliptic expression for "phenomenon represented by a quantitative expression", thus stressing the importance of the relation by which the phenomenon is represented.
- The third position finally emphasizes the unavoidability of interpretive models in knowledge, and assumes "quantitative phenomenon" as actually standing for "phenomenon interpreted as quantitative according to the currently assumed model".

To avoid any bias on terminology, let us denote these positions just as P1, P2, and P3 respectively (although it can be recognized that P1 and P2 are usually called "realist" and "representational" views respectively).

We suggest that a comparative analysis of P1, P2, and P3 can offer some clues for understanding the currently troubled status of Measurement Science (we are alluding in particular to the controversial situation of the 3rd draft edition of the International Vocabulary of Basic and General Terms in Metrology (VIM) [3], in which the existence of two "approaches" is recognized: "The evolution of the treatment of metrological uncertainty from a Classical Approach (CA) to an Uncertainty Approach (UA) necessitated reconsideration of the related definitions in the 2nd edition (1993) of the VIM") and identifying some possible strategies to overcome some of its problems. We also suggest that the distinctions in these positions can be meaningfully illustrated as different orientations to the following questions: *can (and how) Measurement Science be founded? what is the role of truth in measurement?*

2.1. P1 (the "realist" view)

The concept of *true value of a quantity* has traditionally played a fundamental role for the Measurement Science, as attested for example by the derived definitions of *accuracy* ("closeness of agreement between a quantity value obtained by measurement and the true value of the measurand") and *error* ("difference of quantity value obtained by measurement and true value of the measurand"; both the quotations are taken from the VIM [3]).

The position that we have called P1 tends to neglect the role of models: it says "reality is..." instead of "reality is interpreted as...". Such a position can be traced back to the Pythagorean philosophers, who supposed "the numbers to be the elements of all things, and the whole heaven a musical scale and a number" (Aristotle, Metaphysics), and indeed that "any thing that is accessible to our knowledge has a number, since without numbers we can neither understand nor know" (excerpt of Pythagorean school). Since according to P1 "numbers are in the world" (Kepler), and indeed "in measurement, numbers are discovered rather than assigned" [4], "the true value of the quantity X is Y" simply means that Y is an inherent characteristic of X.

It is not easy to understand this position without also embracing its metaphysical assumptions, in particular on the constraint that measurable are only quantitative and continuous properties [4]. Instead of further discussing it, we propose some questions on the general applicability of P1 to measurement, and let them open for further discussions.

1. Whether a physical phenomenon is continuous or not seems to be primarily a matter of Physics, not Measurement Science. Classical examples are electrical current and energy: while before Lorenz/Millikan and Plank they were thought of as continuously varying quantities, after them their discrete nature has been discovered, with electron charge and quantum of action playing the role of ultimate discrete entities. What is the P1 interpretation of these changes in terms of the measurability of such quantities? (they were measurable before the change, no more after; they have never been actually measurable; ...). In more general terms, from the fact that any physical measuring system has a finite resolution the conclusion follows that all measurement results must always be expressed as discrete (and actually with a small number of significant digits) entities: does it imply according to P1 that "real" measurements are only approximations of "ideal" measurements, or what else?

2. The P1 requirement that measurable quantities be quantitative seems to imply a clear-cut threshold between "quantities" and "non-quantities". In reference to the theory of algebraic structures, where such a threshold should be put? Only the field of real numbers is "quantitative"? What about rational numbers? Bounded rational numbers? Are numbers modulus n still "quantitative"? Is the abelian group of integers with addition still "quantitative"? … And, above all, is this choice just based on terminological habits or has some epistemic reasons?

3. Were P1 really supporting the Kepler's view that "numbers are in the world", even considering the easiest example of a quantity such as length the question arises: how many digits has the length of the table on which I am writing now? Surely not infinite, because the very concept of length looses any meaning at the atomic scale. Does this mean that length is not really a "measurable" quantity, or what else?

As a final comment on P1, we would like to quote the explicitly realist (although not *so* realist as P1, of course) standpoint of N.Rescher on this topic: "measurement is more than a matter of mere quantification; only in special cases do quantities actually *measure* something. Quantification in and of itself is no guarantor of objectivity (...); objectivity, after all, does not require quantification" [5]; the epistemic role of measurement is understood when recognizing that measurement is a tool for obtaining and expressing objective and intersubjective information on empirical objects [6], not a tool, or even *the* tool, for quantification.

2.2. P2 (the "representational" view)

In opposition to the idea that "numbers are in the world", the standpoint can be maintained that "numbers are assigned to the nature by ourselves" [7], thus focusing on the relation by which numbers, and more generally linguistic entities, are assigned to portions of reality ("mass is a relation between a body and a number", quoted again from [7]).

In the same perspective the Oxford English Dictionary defines truth as "conformity with fact; agreement with reality", in accordance with the so-called *correspondence theory of truth*, the view that intends *truth as a property of a relation* (variously considered as correspondence, conformity, congruence, agreement, accordance, copying, picturing, signification, representation, reference, satisfaction) *between portions of reality* (various concepts are employed for the relevant portion of reality: facts, states of affairs, situations, events, objects, sequences of objects, sets, properties, tropes) *and informational constructs* (correspondence theories of truth have been given for beliefs, thoughts, ideas, judgments, statements, assertions, utterances, sentences, and propositions. It has become customary to talk of "truthbearers" whenever one wants to stay neutral between these choices) (adapted from [8]). Therefore according to this view what can be true is, in particular, a proposition (a "truthbearer") is its role of representation (a "relation") of a property (a "portion of reality").

It is interesting to note that, despite of this variety, "values" are not listed among the (possible) truthbearers: this suggests that in the idiomatic form "true value of a quantity" P2 considers elliptic the attribution of truth to values, so that "the true value of the quantity X is Y" should be actually interpreted "the proposition 'the value of the quantity X is Y' is true".

The emphasis on the relation between empirical properties and propositions stating measurement results was pursued by developing the idea, proposed in the mid of the XIX century by H.Helmholtz [9], to describe measurement in terms of its formal characteristics: quantities were classified according to the scale transformations under which they are invariant, a work that led to the S.Stevens' theory of scale types [10]. Once embedded in an algebraic framework formalizing measurements as morphisms from empirical properties to symbols, the so-called representational point of view to Measurement Theory emerged [11]. Particularly interesting for our analysis of P2 is the specific characterization of the morphism as discussed by L.Narens [12], who declares that since "the choice of homomorphisms as the basis for the representational theory of measurement has never been adequately justified" he prefers "to change the character of the representational theory a little and consider a scale to be an isomorphism between the empirical or qualitative situation and some mathematical situation. The primary reason for this is that isomorphisms *preserve truth* whereas homomorphisms do not". This highlights that P2 maintains a metaphysical stance on the role of truth in measurement (and indeed "the correspondence theory of truth is often associated with metaphysical realism" [8]): while its basic difference with P1 is the emphasis on the informational intermediation given by the representation, P2 still assumes the empirical properties as the a priori elements on which the scale construction, and then the measurement itself, can be founded. Such properties are assumed to be observable independently of measurement, and thus play the role of the reality in accordance to which the truth of the representation has to be determined: *measurement would not be entitled to determine the truth, but only to preserve it*, by suitably (i.e., in a morphic way) mapping empirical properties to symbols.

This explains, at the same time and for opposite reasons, why S.Stevens was able to assert that "measurement is the assignment of numerals to objects or events according to rule, *any* rule" [13] and why P2 has not been broadly applied in empirical sciences: if the determination of the truth of measurement results is delegated to a prior activity aimed at identifying empirical properties, then *any* representation rule for such properties can be actually thought of as a measurement, but Physics and Engineering will generally refuse to adopt this position, that requires them to take for granted not the empirical methods of measurement but the problematic [5, 13] identification of empirical properties.

3. The "Paradox of foundation" of measurement and a strategy to cope with it

As all empirical sciences were asking measurement to play the foundational role of "protocol of truth" and Measurement Science accepted this function of delegate to deal with "pure data", measurement itself was forced to the paradoxical position of being at the same time the *most empirically objective* operation, because of its institutional tasks, and the *most metaphysically based* one, because of its conceptual foundation on the hypothesis of the existence of true values. We will call this clashing situation the "*Paradox of Foundation"* (PoF) *for measurement*.

It must be recognized that both the technology related to measurement and many areas of Measurement Science (e.g., signal theory) have not specifically suffered from the existence of the PoF, mainly because they do not require to be explicitly supported by foundational topics for their development (as the terminological frequent, and at the same time operationally immaterial, reference to "true values" witnesses: the assumption of existence of a true value is not required to justify the adoption of statistical techniques). It is usual to find textbooks on

Measurement Science and technology trying “to escape” the PoF with the schizophrenic approach of introducing some basics of measurement (in particular the concept of scale and the classification of scale types) in representational terms, i.e., according to P2, and then presenting the applications (e.g., metrological characterization of sensors, calibration and traceability chain, digitalization and digital devices, …) in terms of true values and errors, i.e., according to P1.

Furthermore, while social sciences typically adopt the formal condition of (homo)morphism (and the related results, as the concepts of meaningfulness in scale transformations and admissible statistics) as a criterion to validate candidate evaluations, the usefulness of such a condition for empirical sciences is limited because:

- measuring systems typically implement the homomorphism in their physical structure and behavior, so that in their usage, i.e., during measurement, the condition is in principle autonomously, and automatically, fulfilled; the check of homomorphism is fundamental in scale construction, an operation that is rarely part of the work of people involved in measurement in the context of empirical sciences;
- according to the recommendations of the ISO Guide to the Expression of Uncertainty in Measurement (GUM) [15], any measurement result must be specified as an estimation of both the measurand value and its uncertainty, the evaluation of the latter being a task that must keep into account personal experience, beliefs, and sometimes even ethics, all components that can be hardly formalized in terms of morphic mappings.

Finally, a few hypotheses can be sketched on the strategies by which the two previously analyzed positions have dealt with the PoF. According to the hyper-realism of P1, it is plausible that the PoF is simply not a paradox at all: *by empirical means human beings can be assured of metaphysical facts, and measurement is one of such means*. The P2 approach to the PoF seems to be instead based on a principle of task division and delegation: *measurement can only guarantee the consistency of the representation, whereas the determination of its truth is assumed by other means* (by recalling the conception of truth as correspondence to reality as originally stated by Aristotle (Metaphysics), “to say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, or of what is not that it is not, is true”, and then expressed by Tarski, “the proposition ‘the snow is white’ is true if and only if the snow is white”, according to P2 to establish whether the snow is actually white is not a task for measurement).

3.1. Fundamental paradoxes of measurement

We believe that several evidences related to the practice of measurement can be taken as suggestions for a *radically different standpoint* to escape the difficulties that the PoF arises, so to assume an epistemologically consistent position with respect to the problem of foundations of measurement. Let us mention a few, possibly the most important, of such difficulties [6, 14].

1. Since measurement results depend on the adopted standard, as formalized by the reference to a measurement unit / scale, critical for measurement is the relation that operatively links measuring instruments to standards through a traceability chain, therefore substantially a sequence of calibrations. This could lead to conclude that standards themselves are the "realizations of the true value" for their quantity and then play the role of actual foundations for measurement. On the other hand, even neglecting the uncertainties the any traceability chain unavoidably implies, standards must be indeed realized by National Metrology Institutes, which are in charge of maintaining high quality national standards and typically accomplish this task by means inter-laboratory comparisons (recently formalized in terms of "key comparisons" by the CIPM Mutual Recognition Arrangement [16]). Therefore this claimed "path towards foundations" cannot but include a component of conventionality.

   But can true values be conventional? (the term "conventional true value" has been actually introduced, plausibly as a means to reduce the conflict between the metaphysical load of the concept of truth and the empirical requirements of measurement, and it is still commonly adopted, unfortunately even in official documents such as [17]).

2. It is recognized that the expression of measurement results generally requires the indication of a measurand value and an estimation of its quality, both of them depending for their evaluation on the previous measurement of a given set of influence quantities. Being each of these influence quantities a new measurand, the mentioned dependence should be in principle iteratively applied, with the consequence that a "well founded measurement" would be impossible. The fact that this iterative process is usually stopped at its first step by assuming that the quantities influencing the measurand are not in themselves influenced by other quantities (thus on the hypothesis that they can be "directly measured"), could lead to recognize empirical measurements as approximations of "true measurements".

   But can true measurement ever be performed?

3. While the adequacy of empirical models is controlled by means of measurement, the quality of measurement results depends on the validity of the models used for designing measurement instrumentation and defining measurands. On the other hand, models are valid only within a given scope: the definition of a quantity is always subject to a threshold effect, as the very concept of intrinsic uncertainty makes clear.

The latter issue, the definition of measurands, is critical. Given the usual assumptions that measurement is a foundational tool for empirical sciences, and measurands must be (or, at least: usually are) identified before their measurement is performed, the following consequences can be drawn:

A. science is founded on measurement,

and:

B. measurement is founded on the definition of measurands,

and:

C. the definition of a measurand is typically founded on scientific models,

clearly a circular argument. If a foundation for measurement is looked for, a model is unavoidably found (in its turn requiring measurements for its validation, in their turn implying a model, in its turn ...): as N.Hanson asserted, data is always theory-laden [18].

3.2. P3

The model-based position that we have called P3 shares with P2 the idea that *measurement results are assigned* to measurands, *not determined* [19], because "values" belong to the information, not the empirical, world, and the relations between such two worlds always maintain some conventional component [20, 21]. Furthermore, it recognizes that *truth*, if it can be found out, *is determined, not assigned*. Indeed, the pragmatic acknowledgment of the unavoidable presence of uncertainty in measurement has no necessary consequences on truth in measurement: being uncertain on X does not imply to refuse that the truth on X can be expressed (and therefore P3 does not imply relativism, although it is compatible with it). Uncertainty focuses on the epistemic, model-bound, status of a subject ("I am not sure on X because I am uncertain on it"), as it is manifest by comparing "it is true that X, but he does not know it" to the less understandable "it is certain that X, but he does not know it". While not always in a coherent way, the GUM seems to adopt this position, in particular when stating that "a Type B standard uncertainty is obtained from an assumed probability density function based on the *degree of belief* that an event will occur" [15].

It is important to note that a non-relativistic interpretation of P3 is fully compliant with the realist standpoint so typical among empirical scientists. While an orientation among P1, P2, or P3 is a general, even ideological, issue, in some cases a scientist embracing P3 can accept partial conclusions based on P1 and, of course, P2:

1. when the resolution of the measurand is coarse, the concept of true value can be maintained, as typical in the

case of counting for coarse-grained, well-defined items: the number of human beings in the small room in which I am writing now can be obtained with an uncertainty that is null for any practical purpose, so that such a number can be thought of and dealt with as a true value;

2. when the qualitative knowledge of the properties is involved, some characteristics seem to be independent of any model: for example, while it is recognized that a quantity can be measured in different scale types (let us consider it a weak form of operationism), the distinction between intensive and extensive quantities seems to be so deeply rooted in our knowledge that its model-dependence practically disappears.

We suggest that this *sensitivity to the model resolution* is possibly the main merit of P3:

- if a reference that is considered sufficiently stable for the required resolution is available (for example because the measurand is coarsely defined, or the repeatability conditions of measurement allow acquiring a high number of instrument indications, or the standard available for the calibration is of much higher quality than the measuring system under calibration), then P3 allows to operate with the values obtained from it as if they were *empirical true values*;
- on the other hand, whenever the intrinsic uncertainty becomes empirically relevant, then P3 allows precisely to recognize the model-dependence of the knowledge obtained by measurement.

This shows why P3 empirically generalizes P1 and P2.

4. (Non-)conclusion

Several scientists, significantly coming from different disciplines (for example, although with standpoints and conclusions not fully coincident, G.Bateson [22], E.Morin [23], and F.Varela [24]), are now proposing a "non fundamental" standpoint, according to which no absolute foundation is possible for science, because human knowledge is essentially based on a continuously iterative, try-and-revise, adaptive, autopoietic process, not so different from the way children learn, in which progressively some elements become more and more solid but nothing is definitive. In this view, knowledge not a building to be founded, but *a network of components* sustaining with each other and assuming a meaning only in the context they are contributing to create. Measurement is a critical means to consolidate this network by operating internally to it: therefore it is not amazing that at least some measurands cannot be ultimately defined, so that their usage always requires the reference to a whole network of related knowledge. Truth is not bound to disappear in measurement despite its lost foundational role: this standpoint can maintain it as a target, maybe inspired by the Xenophanes' words, so often quoted by K.Popper:

> The gods did not reveal, from the beginning,
> All things to us, but in the course of time
> Through seeking we may learn and know things better.
> But as for certain truth, no man has known it,
> Nor shall he know it, neither of the gods
> Nor yet of all things of which I speak.
> For even if by chance he were to utter
> The final truth, he would himself not know it:
> For all is but a woven web of guesses.

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