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<b>Contributor Name</b>	<b>Prof. Dr. Luca Mari</b>
<b>Date Submitted</b>	<b>18.6.2002</b>
<b>Word Length</b>	<b>2294 (with abstract, headings and list of figures)</b>

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# **ERRORS IN DIGITAL SIGNAL SYSTEMS**

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## **ABSTRACT**

Characteristic of the concept of digital coding is the hypothesis that physical signals are just carriers for symbols, so that physical transformations of signals are actually dealt with as data processing operations. Correspondingly, measurement systems that process digital signals are metrologically characterized by identifying the main sources of uncertainty / error in reference to the data acquisition (usually including an analog-to-digital converter) and the data processing (also optionally performing data transmission) subsystems.

## **KNOWLEDGE LISTING**

1. Uncertainty and error sources in digital signals
2. List of typical digital signal uncertainties and errors
3. Digital signal uncertainties and errors in data acquisition
4. Digital signal uncertainties and errors in data processing

## **1: UNCERTAINTY AND ERROR SOURCES IN DIGITAL SIGNALS**

Digital systems are adopted today in a broad range of measurement applications. While supported by the current remarkable innovations in microelectronics and the related technologies, the reason of the widespread usage of digital systems in

measurement is grounded on the purpose itself of the operation: to extract and formally express information from the physical {signals} obtained by the systems under measurement.

Traditional measuring systems behave as {transducers} of measurands to quantities directly perceivable by human beings (such as angular deflections of needles on graduated scales), and as such their operation can be integrally described in terms of physical transformations, the interpretation of physical states as information entities being left to observers. In this case, any further data processing (leading to the so-called indirect, or derived, measurement) is accomplished by devices external to the measuring system, if not manually by the observers themselves.

On the other hand, characteristic of the very concept of digital coding is the hypothesis that physical signals are just carriers for univocally recognizable and mutually distinguishable *symbols*, so that physical transformations of signals are actually modeled and dealt with as *{data processing} operations* (i.e., mathematical functions) *among symbols*.

The opposition hardware-software is paradigm of this transition: while analog systems *consist of* their hardware, in the case of digital systems a progressive virtualization of the hardware layer is obtained, from *hard-wired logic systems*, to *microprocessor-based programmable systems*, to the so-called *{virtual instruments}*, whose operation could be even interpreted *as if* their characterizing software layer is executed on an ideal hardware subsystem (see also mm\_405, mm\_566).

Correspondingly to each of these *levels of abstraction*, different issues arise in the metrological characterization of the systems, and in particular in the identification of the typical sources of uncertainty / error, related to both the hardware and the (multiple) software layers. The combined uncertainty  $u_Y$  summarizing the

contributions of such multiple sources  $u_{X_i}$  depends additively on them, as formalized by the {law of propagation of uncertainty}, as recommended by the ISO *Guide to the expression of uncertainty in measurement* (GUM) (the simplified version of such a law is shown here, applicable in the case of statistically uncorrelated sources) (see also mm\_155):

$$u_Y^2 = \sum_{i=1}^N \left[ \frac{\partial f}{\partial X_i} \right]^2 u_{X_i}^2 \quad (1)$$

where  $f$  is the function modeling the relation that links the measurand to its influence quantities. The equation (1) is obtained as a first-order Taylor series approximation of the model function computed in a ( $N$ -dimensional) point assumed “sufficiently closer” to the average values of the quantities  $X_i$  and under the hypothesis that  $f$  is “sufficiently linear” in the neighborhood of such a point. While usually reasonably correct in the case of instrumentation dealing with smoothly varying quantities, these assumptions could be critical for digital systems, in which non-linearities (that sometimes are very strong, such as those manifesting as the consequence of bugs in the software) are common.

Given the fundamental requirement to formalize any measurement result by expressing both a measurand value and an estimation of its uncertainty, the usage of digital signals and systems (particularly if with software control) usually implies to trade off flexibility with complexity.

## **2: LIST OF TYPICAL DIGITAL SIGNAL UNCERTAINTIES AND ERRORS**

While in some specific cases digital systems integrally operate on digitally coded entities (e.g., in some cases of counting, in which the measurand is inherently discrete), they are widely used also in measurement of continuously varying quantities so that a preliminary stage of analog-to-digital conversion is implied. Once such a

transduction has been completed the operations are performed on coded symbols, i.e., on a purely algorithmic basis (see also mm\_137, mm\_404). The results are then fed into a device acting as output transducer which is sometimes required to convert the digital symbols back to analog signals.

As a consequence, a metrological characterization of digital systems involves the analysis of their behavior in reference to such three general components, each of them being affected by specific causes of uncertainties / errors.

\* *The input subsystem* is aimed at acquiring information on the measurand from the environment and, when needed, converting it in digital form. Its general structure includes then a sensor, a {signal conditioning} component, and an {analog-to-digital converter} (ADC, that in PC-based systems is usually part of a data acquisition card: see also mm\_452, mm\_453). Digital signals are obtained as the output of such a subsystem; hence, strictly speaking the input subsystem does not contribute to the budget of system uncertainties / errors related to digital signals. On the other hand, ADC characteristics and behavior significantly influence the quality of the generated digital signals (conceptually definable as the degree of correspondence with the originating analog signals and operatively affecting the possibility to reconstruct them from the converted digital signals) (see also mm\_454).

\* *The data processing subsystem* is aimed at dealing with digitally coded entities to transform them by means of suitably implemented algorithms and / or to transfer them to remote devices. Uncertainties / errors can appear in both hardware and software layers, because of the presence of physical factors modifying the quantity on which the symbols are coded and low quality of algorithms (or their implementations) adopted in the processing of such symbols respectively. The latter issue grows in relevance as the software adopted for metrological purposes becomes more and more

complex, as is the case of {spreadsheets} or virtual instruments. The current developments in this area are particularly important, as witnessed by the emerging applications of pattern recognition, automatic control, and data fusion based on the so-called soft computing paradigm, in which techniques such as neural networks and fuzzy logic inference are used to exploit uncertainty and partial information (see also mm\_86, mm\_424, mm\_427).

\* *The output subsystem* is finally aimed at making the processed data available to users and user devices (e.g., actuators of control systems), while possibly converting such data to a corresponding analog form. At this stage raw data produced by the measuring system must be converted to information meaningful to the intended users and useful to them. The sources of possible uncertainties / errors in the expression of measurement results from the digital signals representing the instrument readings are multiple, all basically *related to the mathematical model of the measurement system* (we will not deal with this topic here: at this regard see mm\_12, mm\_66, mm\_127). While traditionally assigned to human beings, the definition and the metrological qualification of this model is the main task of the knowledge-based intelligent instruments.

### **3: DIGITAL SIGNAL UNCERTAINTIES AND ERRORS IN DATA ACQUISITION**

The digitalization of analog signals usually implies their time and amplitude discretization, the two basic parameters qualifying such operations being the {sampling} rate and the amplitude resolution (also called bit depth) of {quantization}, measured in samples per second and bits respectively (see also mm\_456). Even in the case of an “ideal” behavior of the ADC, the limitations in size of the data storage

devices and in bandwidth of the data transmission channels are sources of errors on the generated digital signals:

\* the sampling theorem assures that the information conveyed by an analog signal is integrally maintained whenever the signal is sampled at a rate greater than twice its bandwidth (for most applications the time interval between samples is kept constant); the usual technique of low-pass (sometimes band-pass) anti-aliasing filtering (see also mm\_397) is in fact a trade-off between two systematic errors: its application allows to avoid {aliasing} effects but removes any information contained in the cut-off portion of the signal {spectrum};

\* the number of intervals (sometimes called channels or cells) in which the amplitude range is subdivided in quantization specifies the {quantizer resolution}, i.e., the length of the binary word coding each sample, and thus establishes the amount of the error introduced by the quantization; in the simplest case of uniform quantization, when all the intervals have the same half-width  $a$ , each sample of amplitude  $x$  is associated with a channel  $i$  whose mid point (dealt with as the reference value to be coded) is  $c_i$  : the {quantization error} is then  $x - c_i$ , corresponding to a maximum quantization error of  $\pm 0.5$  least significant bits (LSBs) and a null average quantization error; here again a trade-off is implied: to reduce the quantization error the bit depth of the code word must be increased (in other terms, to enhance the ADC accuracy its precision must be also increased).

To characterize the actual behavior of a physical ADC some further parameters have to be taken into account, such as (internal and external) {noise}, {settling time}, short-term and long-term {stability} (the former is sometimes called {repeatability}), {offset}, linearity of {gain}, and (in the case two or more signals are acquired at the same time) {cross-talk}. It is usual that the specifications for such parameters are

directly given by the ADC manufacturer as the interval  $\pm a$  that surely (i.e., with probability =1) contains the corresponding values / errors. This is the typical case in which the ISO GUM recommends type B uncertainty evaluations based on uniform probability distributions (see also mm\_155): the corresponding standard uncertainties are then computed as  $a/\sqrt{3}$  and combined by means of equation (1).

#### 4: DIGITAL SIGNAL UNCERTAINTIES AND ERRORS IN DATA PROCESSING

The simplest kind of data processing is the one performed by systems computing the identity function, i.e., producing as their output the same symbols given at their input, as typically behaves an ideal digital transmission channel. In this case the presence of errors (generally caused by noise sources external to the channel) is modeled in statistical terms, by recognizing that for each input symbol  $x_i$  the channel does not deterministically produce an output symbol  $y_j$  but a {conditional probability distribution}  $P(y_j|x_i)$  (for {binary channels}  $x_i, y_j \in \{0,1\}$ , and  $P(0|x_i) + P(1|x_i) = 1$ ). The average value of  $-\log_2(P(x_i|y_j))$ , called {equivocation} and computed from  $P(y_j|x_i)$  by means of the Bayes theorem, represents the average information lost in the transmission process because of errors. From the channel equivocation  $H(X|Y)$  and the source {entropy}  $H(X)$  the {channel capacity}  $C$  is computed:

$$C = \max_X (H(X) - H(X|Y))$$

(2)

a basic informational quantity, measured in bit/symbol (and more usually in bit/s by multiplying it by the rate of symbol transmission over the channel), whose physical grounds are clearly identified in the fundamental relation:



$$C = W \log_2(1 + S/N) \quad (3)$$

where  $W$  and  $S/N$  are the {channel bandwidth} and {signal-to-noise ratio} respectively (see also mm\_133, mm\_136).

In the case the information flowing from the source has a rate lower than the capacity  $C$  of the channel, several techniques can be adopted to reduce the probability of error at the receiver, all based on the introduction of redundancies and aimed at either error recognition or correction (see also mm\_134).

Typical applications of digital signal processing in measurement are digital filtering (see also mm\_411) and DTF / FFT computation (see also mm\_403), but also higher-level operations are now common, e.g., to compute statistical parameters as in the case of DC / RMS measurement. The fundamental parameters qualifying the arithmetic of a processor are its {overflow}, {underflow}, and {roundoff} error thresholds.

In the common case of the {floating-point number} representation (in which numbers are expressed as  $(-1)^a \cdot b \cdot 10^c$  where  $a \in [0,1]$ , the mantissa  $b \in [1,10)$  has a fixed number of digits, and the exponent  $c$  is an integer spanning among two fixed values), the overflow and the underflow thresholds depend on the maximum positive and negative values of the exponent respectively. On the other hand, roundoff errors depend on the number of digits reserved for the mantissa, and are expressed in terms of the *machine precision*, a value generally related to the characteristics of not only the processor arithmetic-logic unit (ALU) but also the adopted software platform or compiler: this is an important source of complexity in the metrological qualification of data processing modules (a commonly implemented reference for the values of these parameters is the IEEE Standard, see Table 1).

Table\_1\_near\_here

The data processing subsystem is usually so complex that instead of identifying all the relevant sources of uncertainty, as it would be required to apply equation (1), a black box solution is sometimes adopted for its metrological qualification: a *{reference data set}* is chosen, containing a collection of sampled input data with the corresponding expected output, such input data are fed into the subsystem, and the results are compared with the references. From the analysis of the obtained error an estimation of the uncertainty of the data processing results is then inferred.

Finally, the contribution of the possible hardware faults (and correspondingly the degree of fault tolerance of the system: see also mm\_488, mm\_492) should be taken into account for a complete metrological qualification of the system.

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Table 1 - Values of machine parameters in IEEE floating point arithmetic

Machine parameter	Single Precision (32 bits)	Double Precision (64 bits)
Machine precision	$2^{-24} \approx 5,96 \cdot 10^{-8}$	$2^{-53} \approx 1,11 \cdot 10^{-16}$
Underflow threshold	$2^{-126} \approx 1,18 \cdot 10^{-38}$	$2^{-1022} \approx 2,23 \cdot 10^{-308}$
Overflow threshold	$2^{128} (1 - \epsilon) \approx 3,40 \cdot 10^{38}$	$2^{1024} (1 - \epsilon) \approx 1,79 \cdot 10^{308}$