Contributor Name	Prof. Dr. Luca Mari
Date Submitted	18/2/2002
Word Length	2040 (with abstract, headings and list of figures)

BMID	Block name (Your block is in bold)
Super Cluster 1 – Understanding the Measurement Problem	The Measurement Process
63	Units
64	Signals, information and knowledge and meaning
65	Semiotics and application in measurement systems
66	**Models of the measurement process**

Retain this section between the red lines in your submission.
It is needed for the editorial and review processing. This section is not published.

MODELS OF THE MEASUREMENT PROCESS

PROF. DR. LUCA P. MARI

UNIVERSITÀ CATTANEO, ITALY

ABSTRACT

The epistemic requirement that measurement be an objective and intersubjective evaluation is empirically fulfilled by adopting measuring systems that include selective and repeatable sensors and traceable standards; any measurement is then performed as a (direct or indirect) comparison to a chosen standard.

From the instrument output the measurement result has to be inferred by means of a process based on the information gathered from the instrument calibration and a measurement system model. Such a process is only plausible in its results, that must be expressed specifying both a measurand value and its estimated uncertainty.

KNOWLEDGE LISTING

1. Measurement as a comparison model
2. The output/input black box model
3. Set-theoretical model
4. Generalized model

1: MEASUREMENT AS A COMPARISON MODEL

Measurement is an operation of data acquisition and presentation, aimed at expressing in symbolic form the information empirically obtained on a system about a quantity, the {*measurand*} (we accept the common ambiguities of calling “measurand” both the system under measurement and the measured quantity, and the latter in both its general and specific forms, e.g., length and length of a given object in a given time). Peculiar to measurement is the requirement of *being objective and intersubjective*, where objectivity implies that measurement results convey information only related to the system under measurement and not its environment, and intersubjectivity requires that measurement results convey the same information to different subjects. As such, these properties appear an ideal target, justifying the efforts to constantly enhance measurement devices and procedures.

To achieve an acceptable degree of objectivity and intersubjectivity measuring systems are adopted, including selective and repeatable sensors and traceable standards. Indeed:

- although human beings are able to directly sense a fair amount of quantities and are well trained to express in linguistic form their perception (e.g., “it is rather cold”, “this is heavier than that”), their statements are affected by subjectivity, i.e., they report information on both the sensed system and the perceiver state; to avoid the influence of the latter, and thus to enhance the objectivity of the operation, the measurand is transduced by a sensing system whose output ideally depends only on the measurand and is unaffected by {influence quantities} and internal imperfections;
- while related to the measurand, the quantity provided by sensors still depends on their specific behavior; as a consequence, distinct sensors, even if perfectly

repeatable, produce different outputs from the same input; furthermore, in many cases the sensor output quantity, appropriate for signal conditioning and for driving presentation devices, is not dimensionally homogeneous to the measurand. The sensor output must be then dealt with *as an {instrument reading}, not a measurand value*. To make the information obtained by the measurement intersubjective a common reference must be adopted, so that measurand values are expressed in comparison to such a standard. Critical is therefore the possibility to trace the readings to the agreed standard, a condition operatively ensured by instrument {calibration}.

The requirement of empirical comparison to traceable standards is so fundamental that can be assumed as distinctive of measurement; if generic scale-preserving evaluations can be formalized as {homomorphisms} from empirical to symbolic {relational systems} (!RS!) (see also mm_59):

Figure_1_near_here

in the case of measurement such mappings are not direct but mediated by the comparison to standards:

Figure_2_near_here

Finally, when primary standards are not directly available:

Figure 3_near_here

Operations 1 and 2 are usually carried out *before* measurement: nevertheless measurement cannot be completed without them and therefore such operations play an essential role for the definition itself of measurement. As a consequence, measurement results must state a measurand value in reference to the adopted standard, usually expressed in the form of a measurement unit (see also mm_63).

2. THE OUTPUT/INPUT BLACK BOX MODEL

It is a well-known fact that different {*methods of measurement*} exist, each of them corresponding to a specific technique to perform the comparison between the measurand and the standard. While some methods require the synchronous presence of the measurand and the standard (e.g., following the paradigm of the @two arm balance@ provided with a set of standard weights: a direct comparison) many others are based on the usage of devices acting as serializers of the comparison, so that a measurement involves (at least) two interactions: standard-instrument and measurand-instrument.

Figure_4_near_here

In its interaction with the measurand the instrument generates an output; a general problem of measurement can be then stated as follows: *from the output of the measuring instrument ("the reading") its input (the state of the system under measurement and its environment) must be reconstructed, and from this state a measurand value must be inferred*.

To cope with this input-from-output inference problem two basic strategies can be in principle followed:

- the analytical model of the measuring system behavior is identified and the obtained {characteristic function} is inverted, so that from the output readings the input signals are computed. Because of its complexity this approach is seldom adopted;
- the system is regarded as a black box and only its input-output behavior is taken into account: the instrument is put in interaction with a set of (known) standard states and the corresponding output readings are recorded; by a suitable

interpolation this collection of couples becomes the so-called {*calibration curve*}, that can be thought of as a mapping from measurand values to instrument readings (see also mm_12); this function is then inverted, so that each instrument reading can be associated with a measurand value.

The interactions standard-instrument and measurand-instrument have therefore a complementary function: while the former is aimed at creating a calibration diagram:

Figure_5_near_here

the latter uses the inverted diagram to find the measurand value that corresponds to the obtained reading:

Figure_6_near_here

To enhance the user-friendliness of the measuring systems it is customary to set up their presentation component so that the data they display are expressed directly in measurand units, i.e., the calibration diagram is embedded into the systems. While always measurement requires calibration information, in these cases one can properly speak of *calibrated instruments*.

3. SET-THEORETICAL MODEL

The sensor behavior, therefore critical for both calibration and measurement, is usually expressed as a {*characteristic function*} formalizing the input-output conversion performed by the sensor itself.

The sensor input, a couple $(x, \overline{w})$ where $x = x(t) \in X$ is the measurand and $\overline{w} = \langle w_1, \ldots, w_n \rangle = \langle w_1(t), \ldots, w_n(t) \rangle \in \overline{W}$ is a collection of further quantities influencing the sensor behavior, is transformed to its output $y \in Y$. Therefore the sensor characteristic function:

$$f : X \times \overline{W} \times T \to Y \qquad (1)$$

takes the measurand $x(t)$, the influence quantities $\overline{w}(t)$ and the current time t, included to take into account possible time-dependent effects, and associate them with the output signal $y(t) = f(x(t), \overline{w}(t), t)$ to which both the measurand ("the signal") and the influence quantities ("the noise") contribute.

This simple formalization allows us to introduce some basic parameters describing the static behavior of a sensor:

- {*sensitivity*}: ideally $x_1 \neq x_2$ implies $f(x_1, \overline{w}, t) \neq f(x_2, \overline{w}, t)$, i.e., distinct measurand values always produce distinct outputs; the ratio $\Delta y / \Delta x$ expresses the aptitude of the sensor to reproduce measurand variations to output values;
- {*selectivity*}: ideally $f(x, \overline{w}_1, t) = f(x, \overline{w}_2, t)$ even if $\overline{w}_1 \neq \overline{w}_2$, i.e., the sensor output is not affected by the variations of influence quantities; the less is the variability of y due to $\overline{w}$ the better is the sensor (therefore selectivity corresponds to non-sensitivity to influence quantities: the relative contribution of the measurand to the output can be formalized as a signal-to-noise ratio);
- {*repeatability*} and {*stability*}: ideally $f(x, \overline{w}, t_1) = f(x, \overline{w}, t_2)$ even if $t_1 \neq t_2$, i.e., the sensor output is not affected by short-term (fluctuations) and long-term (aging) time effects; the less is the variability of y due to t the better is the sensor (a stable sensor does not require frequent re-calibrations);
- {*linearity*}: ideally $y = ax + b$ (where a and b are given coefficients, possibly with $b = 0$), i.e., f is a straight line, the better the actual sensor behavior is approximated by this equation the better is usually considered the sensor (a linear, zero-crossing sensor is calibrated in a single operation, aimed at determining the slope a).

In addition to these static parameters, the dynamic behavior of the sensor is synthesized by parameters such as its frequency response (see also mm_119, mm_121, mm_129).

The technical specifications for sensors usually include some quantitative evaluation for these parameters in the nominal conditions of usage, expressed by the allowed ranges of measurand and influence quantities.

4. GENERALIZED MODEL

The inference process that leads to the evaluation and the expression of a measurand value is always only plausible in its results, and in general nothing can be inferred with certainty about the measurand value. The causes of this lack of certainty are various, and in particular:

- the model of the measurement system has not identified all the relevant influence quantities and one of them has a significant variability, such that the environmental conditions (including human operators) change after the calibration;
- the measuring system is less stable than expected when the calibration procedure was defined, i.e., the instrument would require a re-calibration before its usage;
- the interpolation shape of the calibration curve does not adequately map the actual instrument behavior (e.g., it is significantly non-linear where a piecewise linear interpolation was chosen), so that for some instrument reading subsets the instrument is wrongly calibrated.

All these cases can be formally characterized by recognizing that the certainty implied in the choice of a single-valued association between instrument readings and measurand values is not adequate: in the interaction with the measuring system during calibration, each measurand value generates an instrument reading that should be considered a sample drawn from a whole set of possible readings. Such variability can be formalized according to a set-theoretical model, so that the information obtained in the calibration is expressed by a {*calibration strip*}, in which an interval of possible readings, whose centre and width can be considered as the nominal reading and an {uncertainty interval} respectively, is associated with each measurand value:

Figure_7_near_here

(note the changes of the calibration strip width along the measurand axis, taking into account non-uniformities in the uncertainty evaluation).

As in the previous (certain, and therefore ideal) case, this diagram is used in its inverted form during measurement: for any given instrument reading an uncertainty interval of possible measurand values is obtained together with a nominal value (see also mm_155).

An even more general approach could be adopted by expressing the uncertainty estimation as a standard deviation, and therefore in a probabilistic framework, as recommended by the ISO *Guide to the Expression of Uncertainty in Measurement* (1993) (!GUM!). The *Guide*, based on a recommendation by the International Committee for Weights and Measures (!CIPM!) (1981), states that measurement uncertainty can be estimated on the basis of both statistical and non-statistical methods, and specifies a procedure to combine such components into a {*combined standard uncertainty*}. The set-theoretical formalization can be then regarded as a specialization of this framework: if the combined standard uncertainty is multiplied by a {coverage factor} then an {*expanded uncertainty*} is obtained, thought of as the half-width of an uncertainty interval.

The inherent presence of uncertainty justifies the fundamental assumption that the result of a measurement must state not only a (nominal) measurand value but also its uncertainty estimation.

REFERENCES

CIPM (1981) Proc.-Verb. Com. Int. Poids et Mesures 49, 8-9, 26, 1981 (in French).

ISO (1993) *Guide to the Expression of Uncertainty in Measurement*, International Organization for Standardization: Geneva, Switzerland.

Krantz, D.; Luce, R.; Suppes, P.; Tversky, A. (Vol.1: 1971, Vol.2: 1989, Vol.3: 1990) *Foundations of Measurement*, Academic Press, New York.

See also the volumes of the Proceedings on the International Workshop "Advanced Mathematical Tools in Metrology", in the Series on Advances in Mathematics for Applied Sciences, World Scientific, Singapore, New Jersey, London, Hong Kong (currently published: Vol.1: 1994, Vol.2: 1996, Vol.3: 1997, Vol.4: 2000, Vol.5: 2001)

List of figure captions

Figure 1 – A generic scale-preserving evaluation. [mm_66_fig_1]

Figure 2 – Measurement as a scale-preserving evaluation obtained by the comparison to a standard. [mm_66_fig_2]

Figure 3 – Measurement as a scale-preserving evaluation obtained by the comparison to a standard derived by a primary standard. [mm_66_fig_3]

Figure 4 – The different usages of the measuring systems as comparators. [mm_66_fig_4]

Figure 5 – A diagram with the example of a curve generated by calibration. [mm_66_fig_5]

Figure 6 – The example of an inverted calibration diagram, for usage in measurement. [mm_66_fig_6]

Figure 7 – A diagram with the example of a strip generated by a calibration in which uncertainty has been taken into account. [mm_66_fig_7]