Measurement: knowledge from information about empirical properties

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This paper is devoted to introducing measurement as a process aimed at producing knowledge by attributing values to properties of objects. Some subjects of the paper are: measurement as a bridge from the empirical world to the information world; the meaning of (and therefore the information conveyed by) the key equation measurand = measured value; measurement scales and units; the distinction between qualitative and quantitative properties.

Introduction

Measurement pervades our society and is undoubtedly an integral part of people's everyday life. We use measurement to acquire information to understand, control, and improve what we do and how we do it. In advanced societies, measurement also plays an increasingly vital role in a large number of situations, being essential not only in scientific investigation but also in promoting social relationships and society evolution, for example in sustaining organizational and technological innovations as well as emerging technologies and multidisciplinary research [1]. However, despite its crucial importance in almost every field of human inquiry and endeavor, fundamental concepts about measurement science are still quite vague [2], and often hindered by stereotypes: everybody benefits from the availability of information produced by measurements and most of us do measure, but not everybody is sufficiently knowledgeable about what is actually at stake when a measurement is designed and performed, and its results finally reported.

In analogy to description, judgment by experience, and guess, measurement is a process by which a bridge is designed, constructed, and crossed between the world of empirical bodies, phenomena, processes, … and the world of information [3]. The usual, simplest version of such a bridge is in the relation

$$\text{property of an object} = \text{value of a property} \tag{1}$$

for example

$$\text{length of a given steel rod} = 1.2345\ \text{m} \tag{2}$$

where in fact the length of a given steel rod is a concrete, empirical entity and 1.2345 m is an abstract, information entity (to maintain the discussion as simple as possible, we will omit here all references to measurement uncertainty).

For sure, not necessarily relation (1) results from a measurement, as it could also be the outcome of an opinion or even a guess: hence characterizing measurement as a quantitative process is not sufficient. Rather, measurement is usually expected to be able to provide information worth the public trust because "reliable, traceable and comparable" [4]. This is the main rationale to maintaining a distinction between "in my opinion the value of x is y" and "the measured value of x is y", and thus to socially justifying the resources devoted to perform measurements. Hence, and particularly in our "big data" contexts, in which assessing the quality of information exploited to make decision is becoming a critical feature, we suggest that our society would benefit from a widespread metrological culture [5], [6]. To this purpose the present paper is focused on the very fundamentals of measurement science, some of whose core concepts are discussed here, as driven by the question: what are the entities in relation (1) and what kind of information does it convey?

And therefore: what kind of information does measurement produce?

Objects, properties, and comparability of properties

We know and operate by assuming that in the empirical world there are things, like molecules of water, steel rods, production plants and processes, planets, tigers, companies, and countries. In order to refer to such diverse things, instead of a phrase like "phenomena, bodies, or substances", as in the *International Vocabulary of Metrology* (VIM) [7], we use generically the term "object" for short. An object usually interacts or is made to interact with its environment in multiple ways: we describe each of these ways of interaction as a *property* of the object. For example, a given steel rod can be hard to move because it has a certain mass and can be hard to fold because it has a certain stiffness: that mass and that stiffness are properties of that object (please note that we do not take these as definitions, but only as identification criteria: roughly, an object is thought of as something that has properties, and a property as something that characterizes how an object interacts with its environment). Further, different objects may interact in a similar or the same way with their environment, and therefore can be *compared* with respect to some of their properties. For example, the length of a steel rod and the width of a door are comparable, whereas the mass of a steel rod and the width of a door are not. It is such a comparability, and the related possibility to discover that some objects behave in the same way and some others do not, that make the observation of properties an informative activity.

Given this preliminary understanding, a metrological framework can be grounded on two basic assumptions:

- there are objects with properties;
- for any pair of properties of objects, either such properties are empirically comparable with each other or not, and comparability is an equivalence relation.

Through comparability, the set of the properties of objects is then partitioned into classes of equivalence. Properties belonging to the same class, i.e., comparable with each other, are commonly said to be of the same *kind*, where a name is usually associated to the class itself, like "mass" or "temperature". According to an extensional, and somewhat reductionist, stance, mass is then the set of all masses, temperature the set of all temperatures, and so on.

Each kind of properties can be analyzed in its structure, as induced by the comparison relation. The simplest type of structure is such that any two properties of the same kind are either indistinguishable or not, where such a relation is also assumed to be an equivalence. Accordingly, any class of comparability, i.e., any kind of properties, is in turn partitioned into a set of equivalence classes of indistinguishability, where the situation of different objects having empirically indistinguishable properties is commonly described as "having the same property" (e.g., the same length, or mass, etc). After the preliminary recognition that a given object has a given property, the recognition of the indistinguishability class to which the property of an object belongs is the basic information that can be obtained on that property: the rod not only has *a* length, but it has *a given* length.

However, while the membership of a property to a comparability class is intrinsic, in the sense that there is no ambiguity whether a property is a length, or a mass, etc, the membership of a property to an indistinguishability class depends on the way the objects are compared with respect to their properties. For example, the length of a steel rod and the width of a door could be indistinguishable in a rough observation, but become distinguishable at an enhanced resolution.

Furthermore, relation (1) assumes that the property referred to on its left-hand side is well identified, so that the length of a given steel rod is a uniquely characterized property of a well identified object, but in practice this may be a non-trivial condition to guarantee. For example, we are used to talk about the length and the temperature of a steel rod, but, if such properties are to be identified with a sufficiently high resolution, we should acknowledge that the faces of the rod are not exactly plane and parallel, and the rod is not perfectly thermally homogeneous. A possible conclusion is that there is nothing in the world like *the* length and *the* temperature of that rod, also because at the atomic scale the rod itself ceases to be identifiable as an object. While being aware that a perfect, unique identification of empirical properties is usually not feasible (possible counterexamples are properties that are fundamental constants according to our best theories, like the speed of light in vacuum and the charge of the electron), it is a fact that both in the daily practice and in the scientific activity we keep on dealing with entities like the length and the temperature of macroscopic objects. The good, pragmatic justification of this is that it proves to be effective: even though we know that the rod does not really have *one* length and *one* temperature, sometimes this simplification does not prevent us to make appropriate predictions and decisions about the rod, for example as related to how it would behave if immersed in a cooler fluid etc. Of course, as is for any model, such a simplification comes at the price that some details are neglected: since there is an obvious trade-off between simplification and effectiveness, it is important to acknowledge that the simplified model remains effective until the information conveyed by relation (1) does not become more specific than what the description of the property admits. For example, what we know of the structure of the rod could lead us to consider that the rod can be effectively modeled as having one length, and therefore relation (1) is meaningful, until length is evaluated with a resolution of the tenth of millimeter.

According to the VIM, this is about definitional uncertainty, "resulting from the finite amount of detail in the definition of a measurand": "the practical minimum [...] uncertainty achievable in any measurement of a given measurand", in the sense that using a measuring instrument with a better resolution, would produce values for a measurand that is not the length of the rod anymore, but, say, the distance between two extreme points on the opposite faces of the rod. Definitional uncertainty is then the uncertainty about our possibility to identify the property as it is defined, and as such it depends on both the object and the way we have defined the property.

Scales, values of properties, and types of (kinds of) properties

Relation (1) is a case of indistinguishability of comparable properties, and as such it is in principle applicable to any kind of property. For example, we could discover that individuals x and y are indistinguishable with respect to their blood group (i.e., "they have the same blood group", as mentioned above), and then write:

$$\text{blood group of } x = \text{blood group of } y \tag{3}$$

where the "=" sign is customarily adopted to indicate that the blood of the two individuals belongs to the same group (i.e., indistinguishability class), and that that group is labeled as A in the ABO System:

$$\text{blood group of } x \text{ (and of } y) = \text{A in the ABO System} \tag{4}$$

Both (3) and (4) require a classification criterion of blood groups, but the second one conveys a more sophisticated information, due to presence on its right-hand side of a reference to *a value of a property* (a value of blood group, in this case). As it is clear, values of properties operate as selectors in given classifications: in this example, we first learn how to compare blood groups, then we agree upon a comparison-based classification of blood groups, and finally we assign an information entity as unique identifier (A, B, AB, O) to each class, thus creating a scale, i.e., a mapping from classes of indistinguishable properties to identifiers (someone might object against using the term "scale" about structures that are only classificatory, and not at least ordered: let us say that we are using it by extension).

The construction of a scale generates a value for each property of the considered kind, where in the example (4) the identifier A corresponds to the value (A in the ABO System) and so on. While A is simply an identifier of an equivalence class of blood groups, what is a value like (A in the ABO System) is a more delicate issue. Indeed, the sometimes common position that values are "symbols" to represent properties is not informative, as everything can be used to symbolize everything.

What are values of properties then? To better argue on this issue let us come back to more usual cases of physical quantities. Even still lacking a unit, or a scale, of length, we could discover that

$$\text{length of a given steel rod} = \text{width of a given door} \tag{5}$$

(of course always within the limits of the identification of these properties, as discussed above), and then, when the scale generated by the unit metre is available, that

$$\text{length of a given steel rod (and width of a given door)} = 1.2345\ \text{m} \tag{6}$$

Differently from the value (A in the ABO System), that is only about a classification, the value 1.2345 m reveals the underlying structure of lengths, that may be additively composed ("concatenated" as sometimes it is said) so that, if it is true that length of a given steel rod = width of a given door, then their composition is twice as long as each of them, and so on. This supports the usual interpretation of the value 1.2345 m as what is obtained by taking 1.2345 times the metre, whatever the metre is.

Hence, a new issue arises: what are *units*? Of course, an extensional, reductionist stance can be adopted also in this case, by taking for example the metre as an equivalence class of lengths (how such class is established, and therefore how the metre is defined, is not relevant here: it could be identified (the dates that follow refer to the Metre Convention [8]) (i) as the length of an artifact (from 1889 to 1960), or (ii) as a multiple or submultiple of the length of a natural phenomenon (from 1960 to 2019), or (iii) assuming a given numerical value for a length of a natural phenomenon when expressed in metres (since 2019)). But this simplicity generates quite complex interpretations: for example, really is 1.2345 m the multiple of a class of equivalence of lengths? We believe that the traditional, realist stance is well justified here: the metre is not an equivalence class of lengths but what the lengths in the class have in common, i.e., an abstract length, and of course the same applies to any other unit.

Everything becomes simple and consistent in this way: if a unit like the metre is a length, a value like 1.2345 m is a multiple of a length and therefore a length in turn. Accordingly, relation (1) is *an equation of two properties*, one of them identified concretely as the property of an object and the other identified abstractly as an element of a classification, so that the information actually conveyed by such a relation is that the considered property of the object belongs to an indistinguishability class. In the example about the length of the rod, claiming that the equation is true implies that the length identified as the length of a given steel rod and the length identified as 1.2345 times the length that is the metre are the same length. Basically, this applies also to blood groups, with only some differences due to the fact that the classification of blood groups is not generated by a unit. In the example (4) above, claiming that the equation is true implies that the blood group identified as the blood group of a given individual and the blood group identified as the value (A in the ABO System) are the same blood group. With this broad, encompassing interpretation, relation (1) can be called the *Basic Evaluation Equation* [9], and on this ground the framework of property (or scale) types is developed.

Indeed, relations other than indistinguishability or operations can be often applied to properties of a given kind. For example, some properties can be ordered, like temperatures, whereas there is not a general criterion to order others, like shapes, and only an indistinguishability relation holds for them. As another example, for some kinds of properties, two or more objects can be combined in such a way that the relation between their properties and the property of the resulting object fulfills the conditions of the addition between numbers, as happens for example for lengths and masses in non-relativistic conditions. Thus, in the set of kinds of properties different *types* can be distinguished based on which relations or operations apply to the properties of that kind ("kind" and "type", that in everyday language are often taken as synonyms, have then specifically distinct meanings here). Claiming that a given kind of properties is of a given type means that the properties of that kind are comparable according to the relations and can be combined according to the operations that are characteristic of that type. For example, stating that that the type of mass is ordinal and additive means that masses can be ordered and combined additively.

Stevens [10] set the framework for the types of (kinds of) properties (that in fact he operationally characterized as types of scales), by identifying four types. The first two, that he called "nominal" and "ordinal", assume that the properties of the considered kind are only comparable with each other, as either indistinguishable or distinguishable in the nominal type (as for blood groups and shapes), or also ordered in the ordinal type (as for hardness of minerals in the case of Mohs scale). The other two types, that Stevens called "interval" and "ratio", assume that on the concerned properties some empirical operations are also possible, and this makes them richer in the information they convey but also more complex in their characterization.

A basic issue comes from the traditional distinction between intensive and extensive properties, as exemplified by positions in space (let us say, a one-dimensional space for the sake of simplicity) and lengths. Positions can be operated by difference, and the difference of two positions is their distance, and therefore a length, but cannot be operated by sum, in the sense that there is not an operation between pairs of positions with the features of an arithmetic addition. Lengths can instead be operated by both sum and difference. Exactly the same applies to positions in time, sometimes called "calendar times", and durations: durations can be added and subtracted, whereas positions in time can only be subtracted ("today plus yesterday" is meaningless) and the result is their distance in time, i.e., a duration.

On this matter Stevens focused on the condition that interval scales / properties, like positions in space and in time, do not have an intrinsic zero: a zero can be set (what in physics is done by defining a spatiotemporal frame of reference) but remains conventional. Vice versa, we consider the existence of a zero property as an integral part of what we know of ratio properties (or ratio scales according to Stevens), like length and duration.

However, things are more complex than the possible association intensive = interval and extensive = ratio could suggest, as the historical development of temperature scales shows. Plausibly thought of as only ordinal in a far past, thermometric scales, like Celsius and Fahrenheit, upgraded temperature to an interval property, and in fact such scales do not identify an intrinsic zero, as witnessed by the fact that the temperatures mapped to the identifier 0 in Celsius and Fahrenheit scales are not the same, at the same time recognizing the intensive nature of temperature, such that the temperature of a body obtained by putting two bodies in thermal contact is not the sum of their temperatures but a weighted mean of them. The development of thermodynamic scales, like Kelvin, led to recognize an intrinsic ("absolute") zero temperature, thus upgrading temperature to a ratio property, while maintaining its intensive nature. This shows that the correct association is extensive = additive, instead of extensive = ratio [11].

The four types originally proposed by Stevens are by no means the only ones. Stevens himself later introduced a fifth type, called "absolute", related to properties that are countable and therefore are considered to have both an intrinsic zero and an intrinsic unit, but examples of other types can be found, like in the case of angle amplitude, that is "circularly additive", i.e., additive modulus 2π.

Furthermore, note that the type-related structure of a kind of properties is observed under specified empirical conditions. For example, electrical resistances of resistors combine additively if combined in series, but not in parallel (in a hypothetical world in which only parallel combination of electrical resistances were possible or known, electrical resistance would be supposed to be an intensive ratio property, analogous to temperature). Hence, stating that a given kind is of a given type is a shorthand for something like: given the currently best available knowledge, at least one procedure is known to operate on the properties of that kind according to the type-related structure [12].

Finally, note that this framework does not provide any specification about the distinction qualitative vs quantitative, that thus remains partly conventional. For example, according to the VIM a total order is sufficient for a property to be a quantity, whereas the axiomatic approaches developed from Holder's [13] consider "continuity as a feature of the scientific concept of quantity". In the traditional perspective that assumes being quantitative a necessary condition for a property to be measurable, this diversity is perplexing [14]. As we have seen, Basic Evaluation Equations apply with only minor differences irrespective of property types: while evaluations in algebraically richer scales provide more structural information, our society needs to rely on trustworthy information also on nominal or ordinal properties, by means of "widely defined" or weakly defined" measurements [15]. This is the broader picture we will pursue in the other papers of this series about fundamentals of measurement.

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