preprint of a paper published in:
Metrologia, 55, 716-721, 2018, doi:10.1088/1681-7575/aad8d8

Measurement units as quantities of objects or values of quantities: a discussion

Luca Mari[a#], Charles Ehrlich[b#], Leslie Pendrill[c#]

[a] School of Industrial Engineering, Università Cattaneo – LIUC, Castellanza (VA), Italy

[b] International Legal Metrology Program, Office of Weights and Measures, Physical Measurement Laboratory, National Institute of Standards and Technology (NIST), Gaithersburg, MD 20899-2600, USA

[c] RI:SE Metrology, Research Institutes of Sweden, Göteborg, Sweden

Abstract

Measurement units have historically been defined as quantities (i.e., specific properties) of objects, such as the mass of a particular piece of metal or the length of a particular rod. While the current International System of Units (SI) Brochure endorses this position, the draft 9th SI Brochure proposes to change it, and instead define measurement units as values of quantities. The reason for this proposed change is not provided, but it does not seem plausible that it is related to the redefinition of the SI units in terms of fundamental constants of nature: the very concept of what a unit is does not depend on the concrete way any given unit is defined. This paper is intended to open a discussion of whether measurement units should be defined as quantities or as quantity values, and provides our rationale for maintaining the definition of units as quantities.

1. Introduction

With the forthcoming revision of the International System of Units (SI) based on "explicit-constant" definitions [CGPM 2011], the foundation of metrology will change. Several papers have been published on this topic (see [Milton et al. 2014 and Tal 2018]; the perspective of one of the present authors is in [Mari et al. 2017]), with most of the related discussion rightly devoted to making the new framework consistent with the present best experimental data, as summarized by the Committee on Data for Science and Technology (CODATA) [Newell 2018]. This revised SI will be presented as an updated version of the SI Brochure, currently in its 8th edition [BIPM 2014]. To its credit, the Consultative Committee for Units (CCU), which has done much of the drafting work, has made the several subsequent drafts of the 9th edition of the SI Brochure openly available, the current one being dated 5 February 2018 [BIPM] and approved by the International Committee for Weights and Measures (CIPM) in October 2017 [CIPM 2017].

Alongside the explicit-constant definitions, a change proposed in the latest draft of the SI Brochure significantly affects the understanding of some fundamental concepts of metrology. Where the current 8th edition of the SI Brochure writes [BIPM 2014: 1.1] (emphasis added)

> "The value of a quantity is generally expressed as the product of a number and a unit. *The unit is simply a particular example of the quantity* concerned which is used as a reference, and the number is the ratio of the value of the quantity to the unit."

the draft 9th edition replaces with [BIPM 2018: 2.1] (emphasis added)

> "The value Q of a quantity is expressed by the product of a number $\{Q\}$ and a unit $[Q]$:
>
> $$Q = \{Q\}[Q].$$
>
> *The unit is simply a particular example of the value of a quantity*, defined by convention, which is used as a reference and the number is the ratio of the value of the quantity to the unit."

Hence, together with a new framework for the definition of the units of length, mass, etc., the draft 9th edition of the SI Brochure seems to also bring a *new concept* of unit: instead of *quantities*, units are now presented as *values of quantities*. The reason for such a proposed change is not specified in the draft, is not obvious, and seems to be worthy of some analysis. The present paper intends to highlight these discussions and open the debate to a wider audience, bearing in mind the fundamental import of the issue. The analysis that follows is developed around the

---

[#] The authors are members of the Joint Committee for Guides in Metrology (JCGM) Working Group 1 (GUM) and/or Working Group 2 (VIM). LM is chair of IEC/TC 1; CE is chair of JCGM/WG2; LP is chair of ISO/TC 12. The opinion expressed in this paper does not necessarily represent the view of these Working Groups and Technical Committees, nor of the authors' home organizations.

pivotal relation between quantities of objects and values of quantities, and leads to the conclusion, that we offer for further discussion, that considering units as values of quantities instead of quantities is not only unjustified, but plausibly not the right decision to make.

2. The complex concept of quantity: presentation

The concept of quantity is complex, as our lexical customs reveal: we use the term "quantity" in reference to both entities such as length L ("length is a quantity") and entities such as the length of a given rod a, L_a, ("the length of rod a is a quantity"), and thus leaving to the context the task of removing the ambiguity. Just to emphasize this distinction, we call here *general quantity* an entity such as length and *quantity of an object* an entity such as the length of rod *a* (note that the term "object" in "quantity of an object" refers not only to physical bodies but also to other entities, such as phenomena and substances; hence, this use of the term "quantity of an object" is consistent with what the VIM defines 'quantity' – "property of a phenomenon, body, or substance, where..." – and is a lexical means to maintain an explicit distinction with general quantities).

Hence, the definition that the VIM gives of 'measurement' [JCGM 2012: 2.1]

> "process of experimentally obtaining one or more quantity values that can reasonably be attributed to a quantity"

is to be understood as referring to a quantity of an object: values of quantities, such as $0.123\ m$, are attributed to measurands (i.e., quantities "intended to be measured" [JCGM 2012: 2.3]), such as L_a, that are quantities of objects, not general quantities. This is the relation usually written (we do not take measurement uncertainty into account here) as

> quantity of object = value of quantity

then for example

$$L_a = 0.123\ m$$

We propose the following interpretation of this relation:

- a general quantity length, L, exists, that is independent of any particular object, any measurement, any definition and choice of a unit, and any value;
- an object, the rod a, exists, that has a physical property, its length L_a, which is the quantity of an object, independent of the definition and choice of any unit and therefore of values of quantities. For example, two rods, a and b, can be compared by their lengths, L_a and L_b, and the outcome that a is longer than b, $L_a > L_b$, can be obtained even in a conceptual and experimental context in which no units of length are defined yet; hence *quantities of objects are assumed to exist independently of our knowledge of their values*;
- the value $0.123\ m$ can be identified independently of the existence of any physical object having that length, as the 0.123-multiple of the unit metre. For example, the lengths $\pi\ m$ and $10^{1000000000000000000000000}\ m$ are in principle understood even though our models assert that there cannot be physical objects having such lengths; hence *values of quantities are assumed to exist independently of the existence of objects possibly having such values*;
- while *quantities of objects and values of quantities are conceptually distinct entities*, an experimental activity could lead one to assess that the length of rod a is $0.123\ m$, $L_a = 0.123\ m$, thus producing the information that two conceptually distinct entities are in fact the same.

3. Interpreting equality of quantities of objects and values of quantities: sense and reference

The discovery that conceptually different entities are the same is an example of the knowledge advancement expected from measurement, and a canonical case of the complexity of the relation of equality, which in fact is not identity in all possible respects. In a fundamental paper on this subject, G. Frege proposed the example of the relation between the morning star and the evening star: while conceptually distinct, they were finally discovered to be the same celestial body, the planet Venus [Frege 1892]. Since Frege, it is understood that the expressions "the morning star" and "the evening star" *have different senses* (i.e., the *concepts* 'the morning star' and 'the evening star' are different) *but the same referent* (i.e., the *objects* the morning star and the evening star are the same). This explains why the relation *the morning star = the evening star* is informative, whereas *the morning* *star* = *the morning star* is not. Analogously, this explains why primary school children are asked whether $1+1=2$, but not whether $2=2$: the point is that the expressions "$1+1$" and "2" refer to the same object, i.e., the number 2, but have different senses, and the question posed to children aims at assessing whether such different concepts are recognized as related to the same object.

A more operative way to present the idea of different senses/concepts for the same referent/object is provided by the familiar fact that two functions $f : A \rightarrow B$ and $g : A \rightarrow B$ could be defined by different rules and nevertheless be discovered to realize the same mapping. For example, given $f(x)=(x+1)^2$ and $g(x)=x^2+2x+1$ then $f=g$: they are the same function even though the rules by which they are computed are different. From a mathematical point of view, "a function is not the rule itself, but what the rule accomplishes" [Lawvere 1991].

In the context of ISO standards on terminological works, e.g., [ISO 2009], the distinction between sense and reference corresponds, with minor differences, to the distinction between intension and extension of a concept, and it is maintained also through a careful use of delimiters for terms. Terms referring to terms themselves are delimited by double quotes (e.g., "measurement" is an 11-letter word); terms referring to concepts are delimited by single quotes (e.g., 'measurement' is a key concept of metrology); finally, terms with their usual referents are not delimited (e.g., measurement is a key process of metrology). Back to Frege's example, then:

as terms: "morning star" ≠ "evening star"

(one string starts with "m" and the other with "e", and so on)

as senses / concepts: 'morning star' ≠ 'evening star'

(the concepts / the rules for identifying the referents are different)

as references / objects: morning star = evening star

(a discovery of ancient astronomers)

4. The complex concept of quantity: analysis

The fact that a quantity of an object, like L_a, and a value of a quantity, like $0.123\ m$, can be put in a relation of equality shows that they are entities of the same sort: it is clear that the expressions "the length of rod a" and "the 0.123-multiple of the unit metre" have different senses, and nevertheless the truth of the relation $L_a=0.123\ m$ implies that such expressions have the same referent, i.e., that L_a and $0.123\ m$ *are the same length* (again, we are proposing a simplified interpretation here, which neglects the role of the model of the measurand and the presence of measurement uncertainty). By applying the scheme above, a relation such as $L_a=0.123\ m$ means that

as terms: "L_a" ≠ "$0.123\ m$"

(one string starts with "*L*" and the other with "0", and so on)

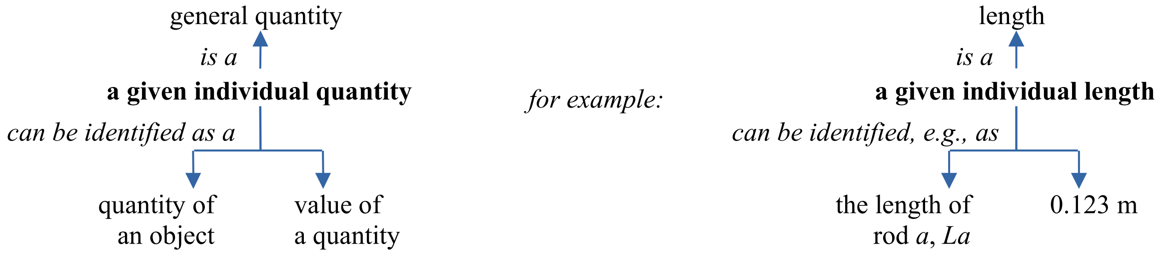
as senses / concepts: 'L_a' ≠ '$0.123\ m$'

(the concepts of the length of an object and of a multiple of a unit are different)

as references / objects: $L_a=0.123\ m$

(a possible outcome of a measurement: they are the same length)

It is then appropriate to introduce a term to designate both quantities of objects and values of quantities as instances of general quantities: we call them *individual quantities* (as does the VIM, [JCGM 2012: 1.1 Note 1]). An individual quantity, for example a given length, can be identified by referring to an object that has that length, or as a (not necessarily integer) multiple of a unit, i.e., as a value of quantity. This can be synthesized in the following diagram:



The fact that we can identify individual quantities as quantities of objects or values of quantities explains the different ways for assessing their possible equality:

- a relation such as $L_a = L_b$ is experimental: the objects a and b are compared to assess whether the lengths they have are indeed the same length (again, uncertainties are not considered here); in this case, no values (nor units, then) are involved;
- a relation such as $0.123\ m = 4.84 \in ¿$ is mathematical: whenever the conversion ratio between the metre and the inch is defined and known, the two values are compared in order to assess whether they are the same length; in this case, no objects are involved (and note that the distinction between sense and reference allows us to appropriately clarify whether $0.123\ m$ and $4.84 \in ¿$ are the same value or not: "$0.123\ m$" and "$4.84 \in ¿$" have the same referent – i.e., $0.123\ m$ and $4.84 \in ¿$ are the same length – but they have different senses; hence, yes, they are the same value if intended as quantities, and no, they are not the same value if intended as concepts);
- a relation such as $L_a = 0.123\ m$ is partly experimental, since it requires the comparison of the object a and another object, and partly mathematical, given that the value $0.123\ m$ can be obtained as the length of the other object, that is constructed as the 0.123-multiple of the unit metre.

Furthermore, note that these differences are not restricted to equality. For example, $L_a > L_b$ is assessed by the experimental comparison of the objects a and b by their lengths; $0.123\ m > 4.74 \in ¿$ is assessed by the mathematical comparison of the two values; and, as above, $L_a > 0.123\ m$ is assessed by a partly experimental and partly mathematical process.

5. Quantities of objects and values of quantities

The acknowledgment that

- quantities of objects and values of quantities are both individual quantities, and that
- the difference between quantities of objects and values of quantities is in *the mode of identification* we adopt

is the basis for justifying why quantities of objects can be modeled as (random) variables that can take values [Mari, Giordani 2012]:

Entities such as L_a, are elements of the world, are individuated in terms of a given object under measurement as quantities of objects that are equated to quantity values.	Entities such as $0.123\ m$, are elements of a set, are individuated independently of any object under measurement as values of quantities that are equated to quantities of objects.

This in turn justifies adopting the usual mathematical notation: the variable is denoted by an uppercase character, say Q_a, and its value by the corresponding lowercase character, q (unfortunately, this clear notation clashes with customary symbols used in physics, such that, e.g., velocity and acceleration are designated by lowercase symbols, v and a respectively). The relation

quantity of object = value of quantity

is then written

$$Q_a = q \qquad (1)$$

(the more usual notation $Q = q$ maintains the ambiguity between general and individual quantities, and the choice of the draft 9th SI Brochure to use the symbol Q for values of quantities contributes to this confusion), thus meaning that

- while we apply different modes of identification for a quantity of an object (e.g., the length of rod a), Q_a, and a value of a quantity (e.g., $0.123\ m$), q,
- we are claiming that they are the same individual quantity.

Precisely this twofoldedness – entities identified differently but for which information has been acquired by means of measurement, leading to conclude that they are equal – makes the relation non-conventional and informative:

still neglecting the role of uncertainty, (1) *is indeed either true or false*. Measurement units play a key role in establishing this.

6. Measurement units as quantities of objects

In the case of kinds of quantities such as length, any value q is built by assuming that a given quantity, taken as the unit, can be iteratively concatenated with itself and partitioned a given number of times. This can be written as

$$q \equiv \{Q\}[Q] \quad (2)$$

where $\{Q\}$ is a number and $[Q]$ is a unit (see also the VIM definition of 'quantity value' [JCGM 2012: 1.19 and Note 1]). It is important to note that $\equiv$ denotes here the is-defined-as relation, and therefore a conventional and non-empirical relation: (2) *is neither true nor false*. In other words, while (1) is empirical, (2) is a definition.

By substituting (2) in (1) we obtain the well-known

$$Q_a = \{Q\}[Q] \quad (3)$$

(see, e.g., [de Boer 1995]) that, in the same sense as (1), is non-conventional and informative, and either true or false (since (3) is not a definition, the fact that individual quantities (Q_a and $[Q]$) appear on both sides of the equation does not imply any circularity; compare this with the circularity that arises if units are assumed to be values of quantities, as discussed in Section 7).

In order to further emphasize the difference between (2) and (3), consider that, differently from (2), (3) is an empirical equality, on which the usual transformations are allowed, in particular:

$$Q_a / [Q] = \{Q\} \quad (4)$$

($L_a/m = 0.123$ in the example above: the ratio of the length of rod a and the metre is 0.123): exactly as for (3), also (4) is a non-conventional and informative equation, which is either true or false. It can be considered the conceptual and historical root of the very concept of *ratio quantity* (i.e., of a general quantity whose instances are invariant by ratio) and consequently of unit. For example, in his *Universal Arithmetick*, Isaac Newton wrote [Newton 1769]

> "by Number we understand [...] the abstracted Ratio of any Quantity, to another Quantity of the same Kind, which we take for Unity"

Hence the number $\{Q\}$ is understood as the ratio of the quantity Q_a and the quantity $[Q]$ of the same kind and taken as the unit. This is exactly what is in (4).

Note, finally, that equations are also possible that involve, e.g., ratios of values of quantities of the same kind

$$q_1 / q_2 = x \quad (5)$$

such as $0.123\ m\ /\ 1.234 \in ¿\ 3.924$,.and ratios of values of quantities with units of the same kind (as in the sentence of the current SI Brochure quoted above)

$$\{Q\}[Q]/[Q] = \{Q\} \quad (6)$$

such as $0.123\ m/\ m = 0.123$.

This is consistent with the basic standpoint that individual quantities can be identified as quantities of objects or values of quantities: depending of the way the concerned individual quantities are identified, assessing their ratio can be then

- an *empirical* activity, as it is required in the case of (4) to establish L_a/m, i.e., how many metres is the length of rod a, or
- a *mathematical* activity, as it is required in the case of (5) to establish q_1/q_2, such that once the ratio of the units of the values q_1 and q_2 is defined and known (e.g., $m\ /\ ¿ = 39.37.$),.then obtaining x in (5) is just a matter of computation.

7. Measurement units as values of quantities: a (preliminary) critical examination

We are not aware of any scientific paper explicitly supporting and explaining the position that units are values of quantities. Hence what follows in this section derives from our recollection of short memos and talks we have been able to read and listen to since the draft 9th SI Brochure was made publicly available.

A first explanation of why someone might take the position that units are values of quantities is purely semantic: if 'value of quantity' is defined, differently from what the VIM does, in such a way to include units as specific cases, then of course units are values of quantities. This seems to be the case of the German standard DIN 1313, of which to our knowledge no official English version is available (what follows is then based on an unofficial translation), that in fact defines ‘unit of measurement’ as “positive quantity value selected by convention” (def. 4.1). This standard defines ‘(scalar) quantity’ (de Größe) as “characteristic, such that for any two values of the characteristic a ratio is defined which is a real number” (def. 3.1) and presents the diameter of a given wheel as an example of the value of a quantity (in note 5 to def. 3.3), where ‘quantity value’ (de Größenwert) is defined as “a value assigned to a distinctive mark of the quantity” (def. 3.3). While extensively used in the standard, the concept ‘distinctive mark’ (de Erscheinungsformen) is not defined, and is only explained in a note as “indicat[ing] to what extent the quantity is ascribed to an object”, where “the quantitative aspect is given by the fact that the distinctive marks can compared multiplicatively, i.e., ratios are defined” (this makes the underlying logic not easy to follow: if distinctive marks can already be compared by ratio, what are such values assigned to them and of which the diameter of a given wheel is an example?). Were the term “quantity” reserved to designate general quantities and the term “value of quantity” intended as referring to any individual quantity, hence both quantities of objects and values of quantities in the terminology we have used here, then the position that units are values of quantities would be correct.

Under the supposition that this is the rationale of the draft 9th SI Brochure, the objection to this explanation is twofold. First, the draft does not provide any hint of a proposal to redefine ‘value of quantity’, about which it only writes how values of quantities are *expressed*. Hence we are left with a conundrum: either we are supposed to maintain the VIM as reference, so that ‘value of a quantity’ is actually *defined* (in the case of ratio quantities) as the product of a number and a unit [JCGM 2012: 1.19 Note 1], but this makes the underlying concept system inconsistent; or we are requested to guess the meaning of the key concept ‘value of a quantity’ only from the indication about how values of quantities are expressed. Second, as we have tried to show above, the distinction between quantities of objects and values of quantities is sufficiently important for measurement to deserve two distinct terms: conflating them to a single term would make a relation such as $L_a = 0.123\ m$ much harder to describe in its content (in discussions with colleagues someone suggested that L_a could be intended as an initially unknown value, that the relation $L_a = 0.123\ m$ makes known, but this is inconsistent with the custom use: in a mathematical relation such as $x = 1 + 1$ the entity x is called “variable”, or “unknown quantity”, or also just “unknown”, but surely not “unknown value”).

Another explanation, not necessarily related to DIN 1313, of the position that units are values of quantities could be based on the argument that values of quantities are multiplications, and in any multiplication the factor 1 can be simplified, so that since for example $1\ m$ is a value then also m, i.e., the unit metre, must be a value.

There are several possible objections to this explanation. The first problem is that it improperly confuses the unit, whose definition requires some reference to the empirical world, and the values that are mathematically obtained from the unit once it has been defined. And indeed the SI correctly defines the metre, the kilogram, etc, not $1\ m$, $1\ kg$, etc. The argument should be then reversed: since units are individual quantities, also values of quantities are individual quantities, exactly as discussed above. Moreover, under the supposition that values of quantities are products of numbers and units, assuming that units are values of quantities leads to a circular definition: in order to know what is a value of quantity we would need to know what is (a number and) a unit, but in order to know what is a unit we would need to know what is a value of quantity.

In synthesis, while this second explanation is just wrong, the first one might be considered more a matter of an incomplete implementation in the draft 9th SI Brochure and of questionable effectiveness of communication. In this view let us conclude our analysis with some further comments.

8. Conclusion

The analysis proposed here suggests that *the characterization of units as quantities is correct* and does not need to be modified. On the other hand, we acknowledge that the distinction *units as quantities* vs *units as values of quantities* is subtle, and sometimes even single words can make a difference in the implied meaning. A key example is Maxwell’s well-known sentence [Maxwell 1873]:

> “Every expression of a quantity consists of two factors or components. One of these is the name of a certain known quantity of the same kind as the quantity to be expressed, which is taken as a standard of reference.

The other component is the number of times the standard is taken in order to make up the required quantity."

As de Boer pointed out, "Maxwell does not speak here of a definition of the concept of physical quantity, but rather gives a factual description of this concept as it is actually used" [de Boer 1995] (see also the analysis in [Mari, Giordani 2012]). Compare now Maxwell's text with the words of Emerson [2008]:

"A value of an (abstract) quantity can be expressed, following Maxwell, as the product of another particular (abstract) quantity (a unit) of the same kind, and a numerical value."

While Maxwell wrote about expressing quantities, thus in reference to equation (3), Emerson is writing about expressing *values* of quantities, thus in reference to definition (2). In the light of this analysis, an improved text for the introductory sentence of the SI Brochure more in line with Maxwell's original intentions, as mentioned in Section 1, might be:

"The quantity Q of an object a (e.g., a phenomenon, a body, or a substance), Q_a, is generally expressed as the product of a number $\{Q\}$ and a unit $[Q]$. The unit $[Q]$ is an example of the quantity Q which is used as a reference, and the number $\{Q\}$ is the ratio of the quantity Q_a to the unit $[Q]$."

This formulation applies also to those individual quantities that are fundamental constants: as any other individual quantity, a fundamental constant Q_a can be identified as the product of a number $\{Q\}$ and a unit $[Q]$. According to the best available knowledge, such a quantity is postulated to be invariant, whatever value it has in a particular unit system. Hence, by fixing a numerical value $\{Q\}$ as the revised SI does, the unit is defined, as $[Q] \equiv Q_a / \{Q\}$, and therefore as a constant quantity in turn.

Describing the number $\{Q\}$ as "the ratio of the value of the quantity to the unit" (as in the sentence quoted from the current SI Brochure) is possible, and in fact corresponds to equation (6). This is correct, of course, but the reference is to a mathematical relation, whereas a foundational presentation of metrology – as the SI Brochure is expected to provide – should also emphasize the empirical components of the process, as in equation (4) and the proposed modified wording above. Our proposed synthesis is then that defining units as quantities (and therefore properties) of objects, not as values of quantities, has several justifications: not only it builds upon a long and well-established tradition, but also, and not less importantly, it avoids circular definitions and contributes to maintain the distinction between measurement, as an empirical process, and the mathematical models that we build of it: measurement can *be formalized* in a mathematical way but *is not* a mathematical process. Hence *it is important to maintain the fundamental distinction between the empirical knowledge about quantities of objects and the mathematical knowledge about values of quantities*. We believe that this is a good argument upon which further discussions on this subject may be grounded.

References

J. de Boer, On the history of quantity calculus and the International System, Metrologia, 31, 405-29, 1995.

Deutsches Institut für Normung (DIN), DIN 1313, Quantities (unofficial English translation), 1998.

W.H. Emerson, On quantity calculus and units of measurement, Metrologia, 45, 134-138, 2008.

G. Frege G, Uber Sinn und Bedeutung eeitschrift für Philosophie und philosophische Kritik, 100, 25-50 (Engl. transl. by M. Black, On Sense and Reference, en.wikisource.org/wiki/On_Sense_and_Reference), 1892.

International Bureau of Weights and Measures (BIPM), SI Brochure: The International System of Units (SI), 8th edition, 2006; updated in 2014, www.bipm.org/en/publications/si-brochure, 2014.

International Bureau for Weights and Measures (BIPM), SI Brochure: The International System of Units (SI), 9th edition, draft dated 5 February 2018, www.bipm.org/utils/en/pdf/si-revised-brochure/Draft-SI-Brochure-2018.pdf, 2018.

International Committee for Weights and Measures (CIPM), Decisions of the CIPM at its 107th session (June 2018), www.bipm.org/utils/en/pdf/CIPM/CIPM2018-Decisions-EN.pdf, 2018.

International Conference on Weights and Measures (CGPM), Resolution 1 of the 24th CGPM, On the possible future revision of the International System of Units, the SI, www.bipm.org/en/CGPM/db/24/1, 2011.

International Organization for Standardization (ISO), ISO 704:2009, Terminology Work – Principles and Methods, 3rd edition, 2009.

Joint Committee for Guides in Metrology (JCGM), JCGM 200:2012, International Vocabulary of Metrology – Basic and general concepts and associated terms (VIM), 3rd edition (2008 version with minor corrections), www.bipm.org/en/publications/guides/vim.html, 2012.

F.W. Lawvere, S.H. Shnauel, Conceptual mathematics – A first introduction to categories, Cambridge University Press, 1991.

L. Mari, P. Blattner, F. Pavese, Improving the understandability of the next edition of the International System of Units (SI) by focusing on its conceptual structure, Measurement, 101, 200-205, 2017.

L. Mari, A. Giordani, Quantity and quantity value, Metrologia, 49, 756-764, 2012.

J.C. Maxwell, A treatise on electricity and magnetism, Oxford University Press, 1873.

M.J.T. Milton, R. Davis, N. Fletcher, Towards a new SI: a review of progress made since 2011, Metrologia, 51, 3, 2014.

D.B. Newell, F. Cabiati, J. Fischer, K. Fujii, S.G. Karshenboim, H.S. Margolis, E. de Mirandés, P.J. Mohr, F. Nez, K. Pachucki, T.J. Quinn, B.N. Taylor, M. Wang, B.M. Wood, e. ehang, The CODATA 2017 values of h, e, k, and N_A for the revision of the SI, Metrologia, 55, 1, 2018.

I. Newton, Universal arithmetick: or, A treatise of arithmetical composition and resolution, https://archive.org/details/universalarithm00wildgoog, 1769.

E. Tal, Naturalness and convention in the International System of Units, Measurement, 116, 631-643, 2018.