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Quantities, Quantification, and the Necessary and Sufficient Conditions for Measurement

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Abstract. Although measurement has been an important component of human activities for millennia, it remains remarkably difficult to provide a fully satisfactory definition of the concept. In part this is due to the fact that measurement is a diverse and dynamic human activity, and takes shape in a wide variety of ways depending on the nature of the subject matter, application, and context. If a definition of measurement is to pay respect to this basic fact, it cannot be so narrowly construed as to apply to only one area of scientific activity (e.g., physics); on the other hand, the definition cannot be so permissive as to trivialize the concept to the point that measurement is not recognizably superior to, for instance, guesses or statements of opinion. One issue at the heart of this tension is the relationship between the concepts of measurement, quantity, and quantification. In particular, it is sometimes argued or assumed either that quantification is a necessary condition for measurement, or that quantification is simply synonymous with measurement. To assess the validity of these positions, the concepts of measurement, quantity, and quantification should be independently defined and their relationships analyzed. In this paper we conduct such an analysis, from both historical and philosophical perspectives, and present the case that quantification is neither necessary nor sufficient for measurement. We conclude by considering how the conceptual separation of measurement and quantification serves to promote more productive and shared understandings of measurement across disciplines.

Keywords. measurement; philosophy of measurement; measure; quantity; quantification; quantitative evaluation

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1. Introduction

Measurement is, and has been for some time, an integral component of a wide range of human activities. In both scientific and lay discourse, measurement is commonly associated with precision, accuracy, and trustworthiness. Yet despite its clear importance and value, it remains remarkably difficult to provide a fully satisfactory definition of the concept of measurement, as the vast array of proposed definitions witnesses (Mari, 2013). In part this is surely due to the very fact that measurement processes have become so widespread: they are now regularly encountered not only in increasingly diverse ways in the physical sciences and engineering, but also (especially within the past century) in the psychological sciences and social research. The scope of activities conducted under the banner of measurement has broadened as the activities that demand precise and trustworthy information have diversified, and it is not always obvious what—if indeed anything—all these ways of measuring have in common with one another.

Some current uses and understandings of measurement are largely motivated based on historical traditions of practice (see for example Michell, 2005 and Sherry, 2011), and therefore it is important to consider a historical perspective when approaching this topic. History helps us understand how the concept of measurement has evolved, and serves as a starting point for rethinking how we can best approach it in the future. We hold that any examination of the characteristics of measurement must be sensitive to how it is understood and used in diverse contexts, and how it adds value to a wide range of human activities. Thus, the task of locating the defining characteristics of measurement, independently of the specific subject matter or application—such that the definition "is broad as it can be without doing undue violence to either the ordinary meaning or the technical meaning of the term" (Savage, 1970, p.158)—is not trivial.

A critical issue at the heart of the many conceptions of and beliefs about measurement is the relationship between measurement, quantities, and quantification[2].

It is commonly agreed that the concepts 'measurement', 'quantity', and 'quantification'[3] are related, though the precise nature of the relationship is less clear. Suppes (2002, p.4), for example, stated that "the primary aim of a given theory of measurement is to show in a precise fashion how to pass from qualitative observations to the quantitative assertions needed for more elaborate

[2]We use the term "quantity" here, as in the International Vocabulary of Metrology (VIM; JGCM, 2012, def.1.1), to refer to a particular sort of property, i.e., the attribute of an entity that possesses quantitative structure (as we will argue below, this requires unpacking). "Quantification" refers instead to the process of quantitative evaluation, understood as the assignment or discovery of a numeric value, such that the minimal characteristic of a quantitative evaluation is that it produces a numerical value (other meanings of "evaluation" and "value", such as the axiological, e.g., "the value of a product", and the ethical, e.g., one's "personal values", are not considered here). Hence one might ask "is x a quantity?" in the same context as "is x greater than 5?" (or possibly "is x greater than 5 m?"), while quantification refers to a process that allows one to ask "are you quantifying?" in the same context as "are you guessing?".

[3]From the methodology of JCGM, as in particular taken from the standard (ISO, 2009), we will use double quotes to delimit words that refer to terms, and single quotes to delimit words that refer to concepts (whereas, of course, terms that refer to objects of the discourse are used without delimiters). Hence, for example, we could write that (the concept) 'measurement' is defined in such and such way so to designate (the object) measurement that in English is denoted as (the term) "measurement".

theoretical stages of science." Statements such as this may be associated with three basic assumptions. The first is that it is in fact possible, at least under some conditions, to obtain quantitative information from qualitative observations. The second is that quantitative information is fundamentally preferable to qualitative information, presumably because quantities offer some privileged access to or insight regarding the property under study, and therefore extracting quantitative information from qualitative observations is a worthwhile endeavor. The third is that measurement is the preferred (or possibly only) method for acquiring such quantitative information.

This understanding is broadly consistent with the views expressed in many classic sources, although details of the expression vary. Consider, for example, the following definitions of measurement[4]:

- "Measurement [is] any method by which a unique and reciprocal correspondence is established between all of some of the magnitudes of a kind and all or some of the numbers, integral, rational, or real, as the case may be" (Russell, 1903, p. 176).
- "Measurement is the process of assigning numbers to represent qualities" (Campbell, 1920, p.267).
- "Measurement [is] the correlation with numbers of entities which are not numbers" (Nagel, 1931, p. 313).
- "Measurement [is] the assignment of numerals to objects or events according to rules" (Stevens, 1946, p.677).
- "Measurement is [...] the discovery or estimation of numerical relations (or ratios) between magnitudes of a quantitative attribute and a unit" (Michell, 1999, p.76).

Though one could note that there seems to be some disagreement regarding what is measured (magnitudes? qualities? entities which are not numbers?), and also regarding the nature of the relationship between that which is measured and numbers (is it one of correspondence? assignment? correlation? discovery or estimation?), the one thing that all these authors agree upon is that, at the end of the day, the output of a measurement procedure is numerical[5].

Broadly, the relationship between measurement and quantification has historically been regarded as one of necessity, one of sufficiency, or one of both necessity and sufficiency, and therefore of synonymy. It is sometimes claimed or believed that:

- in terms of measurement as a process: every quantification is a measurement (i.e., the only way to assign or estimate a value to a quantity is by means of measurement: quantitative evaluation is *sufficient* for measurement); alternatively, or additionally, each measurement is quantification (i.e., only quantities can be measured: quantitative evaluation is *necessary* for measurement);
- in terms of the ontology of measurable properties: each quantity is measurable (i.e., for a property to be a quantity is a *sufficient* condition for it to be measurable); alternatively, or additionally, each measurable property is a quantity (i.e., if a property is not quantitative,

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[4]We are arguing here about the relation between measurement and quantification *from the point of view of measurement*, so as to better understand the nature of measurement itself. Hence we will not discuss positions such as the one according to which "the foundation of quantification is measurement, and any discussion of the nature of quantification must necessarily begin with a discussion of the nature of measurement" (Wilks, 1961, p.5).

5 We suppose that the distinction between numbers and numerals, while in principle significant, can be neglected here.

then it cannot be measured: for a property to be a quantity is a *necessary* condition for it to be measurable).

While the genesis of each of these positions makes some sense when viewed in the appropriate historical context, we argue here that they are all too limiting to be appropriate in the current scientific, technological, and social landscape of measurement:

- claiming that quantification is sufficient for measurement effectively denies the privileged status of measurement—that is, its dependability, accuracy and trustworthiness—relative to trivial activities such as guessing and arbitrary numerical assignment;
- claiming that quantity is necessary for measurement arbitrarily ties a widespread and diverse empirical activity to a particular, limited mathematical concept, namely, that of Euclidean magnitude (or some modified version of it, as we will argue below).

Given that the subject matters of many measurement processes are not quantities in the Euclidean sense, conceptually decoupling measurement from quantification seems to be an important step along the way to achieving an understanding of the acknowledged value of measurement. After all, if the concept of measurement was inextricably tied to the concept of Euclidean magnitude, it would follow that we must either (a) give up on the prospect of having precise, accurate, and trustworthy accounts of non-quantitative aspects of the world, or (b) create a parallel conceptual structure for conducting para-measurement on such aspects of nature.

2. Where not to look for defining characteristics of measurement

Measurement is not a natural entity, existing independently of human beings and discovered by them, but a dynamic social activity with a long history (e.g., Duncan, 1984).[6] The term "measurement" historically acquired its meaning from the way in which it was used; further, in specific fields the concept of measurement has been adapted and formalized in different ways according to the needs at hand. Any definition of 'measurement' so narrow as to include only activities in a specific field (such as physics) would in effect deny that there are relevant features of measurement that may be applied in different contexts, in addition to severely hampering prospects of cross-disciplinary communication. Hence, some conventionality in the definition of measurement is unavoidable, and at least partially different conceptions of measurement can be maintained in different disciplines and contexts.

Nevertheless, it seems reasonable to assume that not just any numerical assignment should count as measurement. After all, measurement is associated with a number of virtues, such as dependability, accuracy, and trustworthiness in both scientific and lay contexts, and it would seem misleading at best to ascribe such properties to any number-generating procedure. Consider, for example, subjective opinions reported by means of numbers (e.g., "I am thirty percent happier today

[6] Partly as a consequence of the fact that measurement is a designed-on-purpose process, the nature and definition of measurement has been interpreted differently across the course of time, in relation to the various technical and social roles that it has taken. Although this is neither a paper about the history of the concept of measurement nor about the history of measurement itself, some historical information will help contextualize our arguments. In particular, we aim to make visible the complex development of the relation between measurement and quantification. In Section 5 these historical sketches will be then interpreted in a philosophical perspective. On this basis, in Section 6 we will conclude that the stance that only quantities are measurable is based primarily on historical convention rather than logical necessity.

than I was yesterday") , guesses (even accidentally correct ones; e.g., "he looks about 1.75 m tall to me"), and the application of completely arbitrary but consistent rules (e.g., Borsboom's [2009] example of taking each person's postal code and dividing by her or his shoe size), all of which produce numerical results but none of which appear to deserve the trust that commonly accompanies measurement. In short, it seems strange to accept that measurement is a privileged process and at the same time that it is defined in so permissive a manner as to include all activities that produce numerical results.

Thus, the relation between measurement and quantitative evaluation deserves to be examined. We propose studying it following two guiding questions (which, for the sake of simplicity, will here be treated as having Boolean answers): (a) is the evaluated property a quantity? and, (b) is it evaluated as a quantity?

The first question relates to claims regarding what the property is, and is thus *ontological*: is being a quantity (or possessing quantitative structure) a necessary condition for the property to be measurable? A positive answer would rule out, in particular, nominal properties (sometimes alternatively called "classificatory", "categorical", or also "qualitative") as entities that can be measured. Such a position is consistent, for instance, with the Aristotelian view that all properties are either quantitative or qualitative. Thus, this issue focuses on the *inputs* of an evaluation process, in terms of the dispositional feature 'being measurable.'

The second guiding question relates to the way in which a property is evaluated, and is thus *operational*: is quantitative evaluation sufficient, and/or necessary, for measurement? Consideration of this issue could be motivated by acknowledging, in particular, that a property that is a quantity might nevertheless be evaluated in manner that does not produce quantitative information. For example, the length of rigid objects may be evaluated according to the condition "is it longer than 1 m and shorter than 2 m?", resulting in a purely binary classification (as in a pass-fail test), even though at the ontological level length remains (putatively) a quantity. Thus, this issue focuses on the *outputs* of an evaluation process, in terms of the categorical feature 'being the result of a measurement.'

Table 1 provides a synthesis of the conceptual framework we are introducing to analyze our topic.

Insert Table 1 here

The four positions, [A], [B], [C], and [D], in the Table are in principle independent, in the sense that one could accept one or more of them and refuse the others. In the balance of this paper we argue that, in fact, all four of them are inappropriate, and the relation between measurement and quantitative evaluation is looser than may be commonly believed (or, stated alternatively, that the concepts 'measurement' and 'quantification' should not be identified so strictly): not all quantitative evaluations are measurements and not all measurements are quantitative evaluations. The first of these points—in opposition to positions [C] and [D] in Table 1—is made in the next section, and the second—in opposition to [A] and [B] above—is made in the one that follows.

3. Is quantitative evaluation *sufficient* for measurement?

Mathematics and physics are intimately intertwined: often, mathematics is presented as the primary language of physics, especially insofar as physics describes how putative quantities such as force, mass, acceleration, and others are interconnected via laws of nature, as given by formulae such as $F = m \times a$. The common assumption is that mathematical equations such as this one can mirror universal truths when interpreted as physical laws[7]. Accordingly, measurement may be seen as the interface between mathematical variables and equations on the one hand, and the empirical world on the other—or, as Campbell (1920, p.267) famously put it, “the object of measurement is to enable the powerful weapon of mathematical analysis to be applied to the subject matter of science” so that “physics […] might almost be described as the science of measurement”. Against this backdrop, Campbell reached the conclusion that “measurement is the process of assigning numbers to represent qualities”.

If Campbell’s definition of measurement is taken at face value, it appears to present measurement not just as simply *a* process, but rather as *the* process of assigning numbers to qualities. This would imply that each and every such process is a measurement, and therefore that quantitative evaluation is sufficient for measurement: if you are assigning numbers, you are measuring. Although it can be argued that Campbell’s definition was predicated on top of additional ontological assumptions and commitments that constrained the scope of measurement (see for instance McGrane, 2015; Michell, 1993, 1999), the idea that the process of assigning numbers constituted measurement would go on to be explicitly embraced by scholars such as S.S. Stevens, who defined measurement as the rule-based assignment of numerals, noting that “[t]he rule of assignment can be any consistent rule. The only rule not allowed would be random assignment, for randomness amounts in effect to a nonrule” (1975/1986, pp. 46-47). This arguably represented a misinterpretation of Campbell’s work (see also McGrane, 2015), but was nevertheless consistent with the most literal interpretation of Campbell’s claim as quoted previously.

As discussed in the previous section, however, equating measurement and numerical assignment effectively denies that measurement has any special virtues in comparison with other activities that produce numerical results, which seems hard to accept given the trust commonly placed in measurement results. In other words, position [D] in Table 1 seems difficult to defend.

However, Campbell and then Stevens were not the only ones to formulate the relationship between quantitative evaluation and measurement as one of sufficiency. Thomas Reid, for example, famously wrote at the beginning of his essay on quantity: “mathematics contains properly the doctrine of measure; and the object of this science is commonly said to be quantity; therefore quantity ought to be defined as what may be measured” (Reid, 1748, Sec.1).This identification of measurement with quantity can be traced back to the Euclidean tradition, as witnessed by the oft-quoted first definition of Book V of the Euclid's *Elements*, which is sometimes thought of as “the earliest contribution to the philosophy of measurement available in the historical record” (Michell, 2005, p.288): “A magnitude is a part of a[nother] magnitude, the lesser of the greater, when it *measures* the greater” (emphasis added; Euclid, trans. 2008)[8]. However, and importantly, this is not a passage about *empirical* measure*ment*, as a verb, but rather, about the *mathematical* definition of *measure*, as a noun, as Bunge (1973) had already noted. In fact, throughout the Elements, ‘measurement’ is never used, and in fact “in the geometrical constructions employed in the Elements [...] empirical proofs by means of measurement are strictly forbidden” (Fitzpatrick, 2008; in his introductory notes to his translation of Euclid’s Elements). This highlights that “measurement” and “measure” should not be dealt with as synonyms: even though they are terms so entangled with daily activities and common discourse that their meaning and relations are often spurious, in scientific contexts the former refers specifically to the process of measuring, not to the structure of

[7] Though we refer to this as a common assumption among physicists, it is not without controversy in the philosophical literature (e.g., Cartwright, 1980).

[8] Here we take ‘quantity’ and ‘magnitude’ to be synonymous.

the input or output entities of a measurement procedure (i.e., properties and property values respectively). For example, the *International Vocabulary of Metrology* (VIM; JCGM, 2012) defines 'measurement' as "the process of *experimentally* obtaining one or more quantity values that can reasonably be attributed to a quantity" (emphasis added)[9]. Indeed, while "measurement" appears to have a relatively well-defined meaning even in everyday use – "the action of measuring", according to the *Oxford English Dictionary* (2010) – "measure" is remarkably polysemic, with several distinct meanings. One of them is particularly important here: a measure is "a quantity contained in another an exact number of times; a divisor". This is indeed the definition that can be traced back to the *Elements*.

Given that the Greek verb for 'to measure' contains the root "metr-", one might conclude that there is a conceptual continuity (or even identity) between the Euclidean 'to measure' and what nowadays is the object of metrology, i.e., measurement. But let us consider another definition by Euclid (trans. 2008), now from Book 7: "A number is part of a[nother] number, the lesser of the greater, when it measures the greater". This definition has the same structure as the one quoted in the previous paragraph: "An *x* is a part of a[nother] *x*, the lesser of the greater, when it measures the greater", and in both cases 'to measure' is used. But while in the first case *x* = magnitude, in the second case *x* = number ("numbers measure one another"), thus showing that here 'to measure' has no necessary experimental connotation. These definitions are followed by several others in which divisibility between numbers is presented in terms of their "measurability": thus, the Euclidean 'to measure' is actually a shortcut for 'to be (an integer) part of'.

For more than two millennia, (Euclidean) geometry was interpreted as intrinsically both a mathematical and a physical science, under the supposition that the physical world *is* Euclidean (e.g., through two distinct physical points one and only physical line crosses; the sum of the internal angles of any physical triangle is 180°, etc.). Non-Euclidean geometries, which disentangled mathematics and physics, were not widely accepted as legitimate until the nineteenth century. Even in the modern era, it could certainly be argued that the concept of measure eventually helped lead to a characterization of empirical quantities and therefore of measurement, but this path is not nearly so continuous as is sometimes claimed, for example by Michell (1996, p.236) when stating that "there are two sides to measurement theory: one side (emphasized in the modern era) at the interface with experimental science, the other side (emphasized in the classical) at the interface with quantitative theory".

A closer look at the historical usages of 'measurement' and 'measure' reveals that, although related, they are distinct concepts. To begin with, the mathematical literature proposes definitions where "the study of measures and their application to integration is known as measure theory" (Cortzen & Weisstein). This shows the conceptual continuity (though not identity) between the traditional, Euclidean concept of measure and contemporary *measure theory*, which – and now the emphasis should be obvious – is not *measurement theory*. And indeed, rereading Reid's (1748) *Essay on Quantity*, doubt emerges as soon as one notices that the term "measurement" is never used at all; instead, the word "mensuration" (defined in the *Oxford English Dictionary* as "the part of geometry concerned with ascertaining lengths, areas, and volumes") appears frequently.

Perhaps most interestingly of all, Hölder's (1901) paper – whose English translation of the title is *The Axioms of Quantity and the Theory of Measurement*, on the basis of which Michell (1999, p.59)

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[9] Interestingly, the VIM avoids using "measure" as a noun (except in the technical, idiomatic term "material measure") to reduce ambiguity, preferring "measurement result" to denote the outcome of the process.

asserts that "we now know precisely why some attributes are measurable and some not: what makes the difference is possession of quantitative structure [i.e., conformity to Hölder's axioms]" (Michell, 1999, p.59) – is opened by this candid sentence: "by 'axioms of arithmetic' has been meant what I prefer to call 'axioms of quantity'" (p.237). Furthermore, aside from the title of the paper, the word "measurement" appears only once in the paper, in a sentence in which "the theory of the measurement" is equated to "the modern theory of proportion" (p.241), thus confirming the purely mathematical nature of the treatment. (One could note then that the paper might have been more appropriately entitled *The Axioms of Geometric Quantity and the Theory of Mensuration*!)[10].

Thus, sentences such as "Euclid's concept [...] explained the place of numbers in measurement [...] and what it is that is being estimated in measurement" (Michell, 2005, p.288), and then, as apparently a direct consequence, "to understand measurement theory, it is necessary to revisit the theory of integration and, particular, Lebesgue measure theory" (Sawyer et al., 2013, p.90), are plainly false. Hence a first conclusion: positions [C] and [D] in Table 1 are untenable; that is, quantitative evaluation is not sufficient for measurement.

4. Is quantitative evaluation *necessary* for measurement?

Having accepted that not all numbers come from measurements, can we assume that all measurements produce numbers? That is, if not sufficient, is quantitative evaluation at least necessary for measurement? Given that, as argued previously, 'measurement' is not a copyrighted concept, one could simply decide to use the term "measurement" to refer only to those experimental processes (with features to be specified) that actually produce numbers, or numbers with units; such a semantic strategy would simply eliminate the problem. But let us investigate the consequences of this option. The position that assumes quantitative evaluation as necessary for measurement is plausibly as follows:

(i) physical laws are written as mathematical equations whose variables represent quantities; and

(ii) only quantities are measurable.

While (i) is a patent fact, it does not necessarily entail (ii), which is explained in the light of the equation:

(iii) quantity = Euclidean magnitude

together with the stipulation that:

(iv) objects of measurement are Euclidean magnitudes.

While (iii) can be assumed as a definition of 'quantity', or just the acknowledgment of a synonymy, as we have just noted (iv) cannot be justified by the Euclidean tradition, which does not deal with any (empirical) "action of measuring". It seems that (iv) is simply a matter of convention or tradition, and thus the question arises: why should measurement be only related to quantities?

From the *Joint Committee for Guides in Metrology*—who is in charge of the development of both the *International Vocabulary of Metrology* and the *Guide to the Expression of Uncertainty in Measurement*—comes a hint that the current conception of measurement is a moving target, instead of being stably bound to the Euclidean tradition. The current version of the VIM maintains that only quantities are measurable (hypothesis (iii) above), but at the same time introduces the concept of (measurable) 'ordinal quantity', "quantity, defined by a conventional measurement procedure, for

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[10] And in fact the alternative but equivalent axiomatization of magnitude presented by Huntington (1902) includes no reference at all to the concept of measurement, as it is presented under the title *"A Complete Set of Postulates for the Theory of Absolute Continuous Magnitude."*

which a total ordering relation can be established, according to magnitude, with other quantities of the same kind, but for which no algebraic operations among those quantities exist" (JCGM, 2012, def.1.26), thus breaking hypothesis (iv): in the Euclidean perspective, the very term "ordinal quantity" would be an oxymoron.

In the lack of a well-established theoretical and metrological structure (Finkelstein, 2003, 2005), the task of studying the structure of a property (with the possible consequence of discovering that it is quantitative) is worthwhile, but in principle it should be acknowledged as a distinct task from the one of measuring the property itself.

As an example of the critical relation between quantification and measurement, consider the case of (simultaneous) "conjoint measurement" (Luce & Tukey, 1964), as in the sentence: "a demonstration that intelligence, say, satisfies (or fails to satisfy) the axioms of conjoint [quantification] would complete the scientific task of quantification for intelligence", taken from (Sherry, 2011), where the original term "conjoint measurement" has been replaced by "conjoint quantification". Michell (2005) correctly notes that "conjoint measurement" refers "not so much [to] a method of measurement [but to] a context within which indirect evidence for quantitative structure could be collected". Were the mentioned replacement accepted, the sentence would be more or less tautological. The original sentence is meaningful only if 'measurement' and 'quantification' are assumed as, first, in principle distinct concepts and second, as actually coincident. As we have shown, there does not appear to be an a priori reason to make such assumptions; in other words, positions [A] and [B] in Table 1 unwarranted and needlessly restrictive.

5. The concept of quantity and philosophies of measurement

Up to this point we have attempted to explore the positions that quantitative evaluation is necessary and/or sufficient for measurement, and to propose our own comments, in as philosophically agnostic a manner as possible. However, given that these claims are often stated in terms of or influenced by particular philosophical stances, and given also that our own position on measurement inevitably involves philosophical commitments, it is worth exploring the implications of two major undercurrents in philosophical thinking about measurement, i.e., empiricism and realism. Our treatment here is necessarily brief (but see Maul, Torres Irribarra & Wilson, 2013).

Empiricist philosophical stances generally emphasize a commitment to direct observation as the basis for knowledge. Representational measurement theories (Krantz, Luces, Suppes & Tversky, 1971), operationalism (Bridgman, 1921; see also Boring, 1923), and the writings of S.S. Stevens (e.g., 1946) may all be considered empiricist approaches to measurement, in that they characterize measurement in terms of the manner in which numerical assignments are derived from observable relations. The idea that quantification is sufficient for measurement can be formulated as a direct corollary to operationalism, insofar as operationalism is commonly interpreted as the stance that measurement is nothing more than the results of applying a particular procedure ('measurement by fiat', according to Torgerson, 1958). As was discussed previously, an extreme version of this position implies that any procedure of numerical assignment can be considered measurement, with the sole exception of random assignment (Stevens, 1975/1986). Others have argued, and we agree, that this trivializes the concept of measurement (Borsboom, 2005; Frigerio, Giordani & Mari, 2010; Michell, 1990; Savage, 1970); certainly, if one accepts that measurement is an activity that seeks to gather accurate, dependable and trustworthy knowledge, it can easily be seen that there are instances of rule-based number assignments that have nothing to do with the acquisition of such kind of knowledge.

By contrast, *realist* philosophical stances on scientific inquiry emphasize the commitments that (i) there is a (single) natural world; (ii) scientific claims about the world are to be taken as possessing truth-values, and (iii) so interpreted, true scientific claims constitute knowledge[11] of the world. There are a range of forms of realism about measurement (e.g., Borsboom, 2005; Michell, 2005; Trout, 1998); on at least one reading, the implication of realism for measurement is that whether or not a property is a quantity is a mind-independent fact about the way the world really is (e.g., Michell, 2005); thus, this interpretation of realism about measured properties might lead to the conclusion that a property must be a quantity in order to be measurable.

Empiricism is motivated by the intuition that the preferred method of acquiring knowledge is through observation and experience, while realism is motivated by the intuition that that scientific inquiry seeks to gain knowledge about the state of affairs in the world. Neither of these intuitions contradicts the other in principle. On the contrary, the connection between the state of affairs in the world and the outcomes of a measurement procedure is not in itself compromised by the fact that we choose to privilege certain contrast classes, levels of explanation, methods of summarization, and modes of description. Indeed, to the extent to which some modeling activities are acknowledged as unavoidable and inherent components of measurement[12] (see, e.g., Frigerio, Giordani & Mari, 2010; Tal, 2013), simplification, abstraction, and idealization are not only to be expected, but are themselves critical determinants of the extent to which the results of the process are interpretable and usable in the intended manner. This, then, implies that the criteria for success in any given measurement activity are most appropriately framed in terms of the extent to which the activity yields results of value in the relevant context—and it should not be expected a priori that, for example, all contexts require that a measured attribute be structured quantitatively in the sense given by Michell (2005) and Hölder (1901), as described previously.

For example, suppose we wish to acquire knowledge about the extent to which students have mastered a set of concepts related to statistical reasoning. We design an assessment of this property of students ("knowledge of statistical reasoning"), based on our best available theories of learning and cognition. We then employ statistical models such as the Rasch model (Rasch, 1960; see also Mari & Wilson, 2014), which treat the measured property of students as a continuous quantity, to test hypotheses regarding both cognitive theories and the assessment instrument (see Wilson, 2004, for examples of how this might be done), and to represent and communicate information about differences amongst students in the extent to which they have mastered statistical reasoning. Engaging in this process may yield outcomes of value for a range of activities: we may advance our collective understanding of how learning works, communicate accurate and dependable information about students to educators and other stakeholders in an efficient manner, and suggest further avenues of exploration both for educational practice and research in educational psychology. Thus, in this example, information has been acquired, represented, and communicated using the logic and language of measurement, to scientifically and practically productive ends, using a model that treats knowledge of statistical reasoning as a continuous quantity. Notably, it was *not* necessary to assume that the quantitative structure of differences in values of this property was an unconditional fact about the natural state of affairs in the world—rather, the treatment of this property as a continuous quantity was a modeling choice, with a justification that is context- and use-specific.

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[11] Where knowledge is taken here to mean a justified true belief (see, e.g., Ichikawa & Steup, 2014).

[12] As illustrated, for example, by the literature on the evaluation of definitional uncertainty (e.g., JGCM, 2012, def.2.27).

6. What, then, is measurement?

In this paper we have developed an argument along two complementary lines. First, we have argued that quantities are, historically and conceptually, tied with measures, not measurement, and 'measure' and 'measurement' are different concepts: what is true of measures should not be uncritically applied also to measurement. Second, the structure of measurement is generally independent of the possible quantitative structure of the property under consideration: the stance that only quantities are measurable is based on historical convention rather than logical necessity. We have focused on a critique of the relation between measurement and quantification in light of the prevalent stereotypes in the field, thus mainly developing the *pars destruens* of the enterprise of rethinking an encompassing understanding of measurement.

The arguments developed in this paper suggest that a productive understanding of measurement—namely, one that accounts for the myriad of contexts in which the concept is applied, while respecting its value as an accurate, dependable and trustworthy tool for gaining knowledge—need not be shackled to the notions of quantity and quantification. It may well be that for some disciplines and under some contexts of application that the conceptualization and practice of measurement does in fact overlap strongly with quantification, but the case laid out in this paper emphasizes that it is not reasonable to demand that 'quantity' or 'quantifications' be either universally necessary or sufficient conditions for measurement.

One obvious consequence of this proposed conceptual decoupling is that categorical and ordinal properties may appear to be candidates for measurement, in addition to quantities.[13] Although the analysis laid out in this paper does not constitute a positive argument for their inclusion within the scope of accepted instances of measurement, it does indicate that there seems to be no good *a priori* reason for disallowing processes that generate categorical or ordinal evaluations from the realm of measurement solely based on the mathematical form of their outputs. An ecumenical understanding of measurement could then consider incorporating the study of properties that are best conceptualized as taxonomies (e.g., personality types or genes) and orders (e.g., levels of performance). After all, if measurement is an activity aimed at the acquisition of actionable, accurate, and dependable knowledge, it may be useful to consider that it could in principle applied in a variety of ways to a variety of properties with which we engage, which may include non-quantitative attributes, rather than arbitrarily segregating such activities purely on the basis of the mathematical form of their results—and, perhaps, forcing some fields of study to develop entirely independent bodies of literature around such practices.

It may be that the hallmark characteristics of measurement are best thought of as located in the structure of the process, either instead of or in addition to particular features of its outputs (such as whether information is quantitative, or can be represented on a ratio scale) or inputs (such as whether the property being evaluated is a quantity). In our view, when one claims to engage in 'measurement activities', one is claiming that one is attempting to develop methods of obtaining

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[13] As mentioned in Section 4, the interpretation of categorical and ordinal properties in the context of metrology, which is traditionally focused on quantities, is a complicated issue. To take the VIM as an example once again, its second edition simply omitted any reference to ordinal or categorical entities, thus implicitly suggesting that such entities are not measurable. In order to take the fact into account that ordinal evaluations are sometimes presented as measurements – the most common example being Mohs scale of hardness – the third edition of the VIM extended the scope of measurability by including ordinal entities, under the term "ordinal quantity". On the other hand, measura*bility* is a conceptual issue, not merely a lexical one.

high-quality information[14] about a property. It may turn out that the property under investigation could actually be quantitative in the strict sense of empirically conforming to Hölder's (1901) axioms of quantity, or could be treated as quantitative within a specified context of application and according to the available knowledge of it, as exemplified in the previous section, in which case the particular method of obtaining information about the property may indeed be to discover or estimate ratios of magnitudes of quantity relative to a standard unit. However, whether or not variation in a given attribute is structured quantitatively (or can be modeled as such) may not be known from the outset -- for example, it was only discovered after centuries of study that temperature could actually be evaluated in a quantitative way (Sherry, 2011); are we then to say that all pre-1760 work on discovering differences in temperatures do not count, *a priori* and independently of how they were performed, as measurement activities? Further, it may turn out that the property cannot not be reasonably considered quantitative at all, and in our view it seems arbitrary and unnecessary to then disallow the use of the term 'measurement' when referring to the acquisition of high-quality information on that property. From this perspective, it seems preferable to consider the extent to which a particular process is more or less measurement-like in terms of the quality of the information it provides, which reframes the question of how to understand what measurement is or is not as being expressed along a gradient rather than as a Boolean classification. We have argued elsewhere in favor of examining measurement procedures in terms of the extent to which structural features of the process guarantee object-relatedness and subject-independence ("objectivity" and "inter-subjectivity" for short) of the results (e.g., Mari, Carbone & Petri, 2012). This could be regarded as the beginnings of the *pars construens*, to complement the *pars destruens* developed in the present paper; nevertheless, there is surely much more systematic work to be done on this topic.

We certainly agree with Michell (e.g., 2005; 2008) that the a priori assumption that a given property is quantitative can be problematic (one might even say "pathological," if and when this assumption is not acknowledged or recognized). Part of the very point of measurement activities, in our view, is to test hypotheses about properties, and in so doing learn more about how facts about them can be (accurately *and* usefully) represented. There are two points on which we depart from Michell: (a) we do not think that quantitivity is necessarily a mind-independent feature of properties, and (b) we do not think that quantity—mind-independent or not—should be considered a necessary condition for measurement.

Measurement is a complex and challenging endeavor. In our view, the claims that quantification is sufficient or necessary for measurement both trivialize the concept of measurement: the former position in effect denies that measurement is an epistemic activity, and the latter arbitrarily ties an empirical activity (measurement) to a specific mathematical concept (Euclidean magnitude). Conceptually decoupling measurement from quantity helps pay respect to the fact that measurement is a dynamic activity aimed at the acquisition of knowledge, and based on this decoupling we can explore alternative conceptual bases for a definition that is not bound by these preconceptions, potentially taking many different forms depending on the domain of application.

[14] The definition of what constitutes high-quality information is an issue beyond the scope of the present paper, but in a general sense the appraisal of quality generally involves evaluation of the degree to which there is a theoretical basis for the procedures that generated the information, the extent to which the results are stable and can be reproduced, and the usefulness of the information in assisting the accomplishment of tasks or the answering of questions of interest. In this paper we have alluded to the idea of high-quality information in terms of information that is accurate, dependable and trustworthy.

7. References

Boring, E. G. (1923). Intelligence as the tests test it. *New Republic*, 35-37.

Borsboom, D. (2009). Educational measurement, 4th edition: Book review. *Structural equation modeling, 16,* 702-711.

Bridgman, P.W. (1927). *The logic of modern physics*. Macmillan: New York.

Bunge, M. (1973). On confusing 'measure' with 'measurement' in the methodology of behavioral science. In M. Bunge (ed.), *The methodological unity of science*. Dordrecht-Holland: D. Reidel, 1973.

Cartwright, N. (1980). Do the laws of physics state the facts? *Pacific Philosophical Quarterly, 61*, 75–84.

Cortzen, A., & Weisstein, E.W. *Measure*. In: MathWorld – A Wolfram Web Resource, http://mathworld.wolfram.com/Measure.html.

Duncan, O. D. (1984). Notes on social measurement: historical and critical. New York: Russell Sage Foundation.

Euclid's Elements of geometry, the Greek text of J.L. Heiberg (1883–1885) edited, and provided with a modern English translation, by Richard Fitzpatrick, 2008, ISBN 978-0-6151-7984-1, http://farside.ph.utexas.edu/Books/Euclid/Euclid.html.

Finkelstein, L. (2003). Widely, strongly and weakly defined measurement. *Measurement*, *34*, 39-48.

Finkelstein, L. (2005). Problems of measurement in soft systems. *Measurement*, *38*, 267-274.

Frigerio, A., Giordani, A., & Mari, L. (2010). Outline of a general model of measurement. *Synthese, 175*, 123–149.

Giordani, A., & Mari, L. (2012). Property evaluation types, *Measurement, 45*, 437–452.

Huntington, E. V. (1902). A complete set of postulates for the theory of absolute continuous magnitude. Transactions of the American Mathematical Society, 3(2), 264–279.

Ichikawa, J., & Steup, M. (2014) "The Analysis of Knowledge", In E. Zalta (ed.), *The Stanford Encyclopedia of Philosophy.* Retrieved from http://plato.stanford.edu/archives/spr2014/entries/knowledge-analysis

ISO 704:2009, Terminology work – Principles and methods, International Organization for Standardization, 3rd ed., 2009.

Krantz, D. H., Luce, R. D., Suppes, P., & Tversky, A. (1971). *Foundations of measurement, vol. 1: Additive and polynomial representations.* New York: Academic Press.

JGCM (2012). *International Vocabulary of Metrology (VIM) – Basic and General Concepts and Associated Terms* (2008 edition with minor corrections), Joint Committee for Guides in Metrology, http://www.bipm.org/en/publications/guides/vim.html.

Luce, R.D., & Tukey, J.W. (1964). Simultaneous conjoint measurement: A new type of fundamental measurement. *Journal of Mathematical Psychology, 1*, 1–27.

Mari, L. (1999). Notes towards a qualitative analysis of information in measurement results. *Measurement, 25*, 183–192.

Mari, L., Carbone, P., & Petri, D. (2012). Measurement fundamentals: A pragmatic view. *IEEE Trans. Instr. Meas, 61*, 2107–2115.

Mari, L. (2013). A quest for the definition of measurement, *Measurement, 46*, 2889–2895.

Mari, L., & Wilson, M. (2014). An introduction to the Rasch measurement approach for metrologists. *Measurement*, *51*, 315-327. doi:10.1016/j.measurement.2014.02.014

Maul, A., Wilson, M., & Torres Irribarra, D. (2013). On the conceptual foundations of psychological measurement. *Journal of Physics: Conference Series, 459*. doi: 10.1088/1742-6596/459/1/012008.

McGrane, J. (2015). Stevens' forgotten crossroads: The divergent measurement traditions in the physical and psychological sciences from the mid-twentieth century. *Frontiers in Psychology, 6:431. doi: 10.3389/fpsyg.2015.00431.*

Michell, J. (1999). *Measurement in psychology: A critical history of a methodological concept.* Cambridge University Press Cambridge, England.

Michell, J. (2005). The logic of measurement: A realist overview. *Measurement, 38*(4), 285–294.

Michell, J. (2008). Is psychometrics pathological science? *Measurement: Interdisciplinary Research & Perspectives, 6(1),* 7–24.

Oxford Dictionaries, Oxford University Press, 2010, http://oxforddictionaries.com.

Putnam, H. (2000). *The threefold cord: Mind, body and world*. New York City: Columbia University Press.

Rasch, G. (1960/1980). Probabilistic models for some intelligence and attainment tests. Chicago, IL: University of Chicago Press.

Sawyer, K., Sankey, H., & Lombardo, R. (2013). Measurability invariance, continuity and a portfolio representation. *Measurement, 46*, 89–96.

Sherry, D. (2011). Thermoscopes, thermometers, and the foundations of measurement. *Studies in the History and Philosophy of Science, 42,* 509-524.

Stevens, S. S. (1946). On the theory of scales of measurement. *Science, 103(2684),* 677–680.

Stevens, S. S., & Stevens, G. (1986). Psychophysics: Introduction to its perceptual, neural, and social prospects. New Brunswick, New Jersey: Transaction. (Original work published in 1975).

Suppes, P. (2002). *Representation and invariance of scientific structures*. CSLI Publications.

Tal, E. (2013). Old and new problems in the philosophy of measurement. *Philosophy Compass, 8*, 1159-1173.

Torgerson, W. S. (1958). Theory and methods of scaling. New York, NY: Wiley. (Cit. on pp. 32, 38, 82).

Trout, J.D. (1998). *Measuring the intentional world: Realism, naturalism, and quantitative methods in the behavioral sciences.* New York: Oxford University Press.

Wilks, S.S. (1961). Some aspects of quantification in science. In H. Woolf (ed.), *Quantification: a history of the meaning of measurement in the natural and social sciences*. Indianapolis: Bobbs-Merril.

Wilson, M. (2004). *Constructing measures: An item response theory approach*. Routledge Academic, 2004.

Table 1. Positions on the necessity and sufficiency of quantity for measurement

Question:	is the evaluated property a quantity?	is the property evaluated as a quantity?
Type of question:	*ontological*	*operational*
Focus of question:	the *inputs* of the evaluation process	the *outputs* of the evaluation process
In terms of:	the *dispositional* feature of '*being measurable*'	the *categorical* feature of '*being the result of a measurement*'
Implications of necessity:	[A] supposing that a property being a quantity is *necessary* for it to be measurable implies that all non-quantitative properties are a priori not measurable, so that nominal and ordinal properties cannot be measured, whatever kind of evaluation process is designed and operated	[B] supposing that a property being evaluated quantitatively is *necessary* for measurement implies that only quantity values can be the result of a measurement, whatever kind of evaluation process is designed and operated
Implications of sufficiency:	[C] supposing that a property being a quantity is *sufficient* for it to be measurable implies that all quantitative properties are a priori measurable, and thus measurability is independent of any experimental condition	[D] supposing that a property being evaluated quantitatively is *sufficient* for measurement implies that all quantitative evaluations are measurements