

## A quest for the definition of measurement

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### Abstract

Since in the scientific and technical literature multiple, sometimes incompatible, definitions of ‘measurement’ can be found, identifying a single conceptual framework is a significant target for measurement science, towards a generalized concept of measurement, in compliance with the notion of widely-defined measurement proposed by Ludwik Finkelstein. This paper introduces the subject with a structured review of some paradigmatic positions and then proposes to characterize measurement as an evaluation process able to produce objective and inter-subjective information on the measurand. A justification is given that this standpoint encompasses the evaluation of both physical and non-physical properties.

### Keywords

measurement science, measurement, measurable quantity, objectivity, inter-subjectivity

### 1. Introduction

The problem of establishing the definition of ‘measurement’ has surely something to do with conventions, and indeed it is common today to be skeptic about the existence of “true meanings” for terms and “true definitions” for concepts. As a consequence, any related discussion might be assumed as mainly of interest for the construction of a lexical system, a task customarily considered outside the scientific endeavor. The remark of the multiple, sometimes incompatible, definitions of ‘measurement’, widespread in the scientific and technical literature, might be simply assumed as the proof that measurement is a many-faceted activity, and that this multiplicity is somehow irreducible.

On the other hand, the question *what is measurement?* is compelling for at least two reasons.

First, the fundamental nature of measurement, acknowledged to be a (or even *the*) basic process to acquire and formally express information on the world, makes it an inter-disciplinary tool, thus emphasizing the usefulness of a global understanding of the basic and general concepts (hence not only ‘measurement’, but also, e.g., ‘measurand’, ‘measurement result’, ‘uncertainty’, ‘accuracy’, ...), where the relation among concepts and the associated terms should be as much shared as it is possible<sup>1</sup>. Consider the example of properties such as the quality of industrial products, the complexity of software systems, the user satisfaction about social services, the individual attitude over given tasks / jobs, ... It is a fact that all of them are routinely evaluated, i.e., the information available on them is represented by means of values, usually numbers. But are such value assignments (“evaluations” for short henceforth) specifically measurements, as it is usually claimed? Or are they only, e.g., “subjective evaluations”, in the form of guesses, assessments by experience, ...? And what is the nature of the so-called soft measurement [2], [3], or weakly-defined measurement, or widely-defined measurement [4]? In these terms, the problem loses most of its conventionality, at least because only in the case of measurements the sometimes significant resources required to accomplish such evaluations would be accepted. The issue is about the “special reliability” (just to use a very generic term for now) of measurement, a feature which has nothing to do with lexical issues and whose justification eventually requires a common understanding of the concept.

The second general reason of interest for the question ‘what is measurement?’ is that an investigation on this matter reveals a strong, systematic correlation between the conceptions of measurement and the underlying

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1 This is precisely the purpose of the *International Vocabulary of Metrology – Basic and general concepts and associated terms* (VIM3) [1], a guidance document published by the Joint Committee for Guides in Metrology (JCGM), an inter-organizational committee currently composed of eight leading international organizations: International Bureau of Weights and Measures (BIPM), International Electrotechnical Commission (IEC), International Federation of Clinical Chemistry and Laboratory Medicine (IFCC), International Laboratory Accreditation Cooperation (ILAC), International Organization for Standardization (ISO), International Union of Pure and Applied Chemistry (IUPAC), International Union of Pure and Applied Physics (IUPAP), International Organization of Legal Metrology (OIML). The VIM3 “is meant to be a common reference for scientists and engineers – including physicists, chemists, medical scientists – as well as for both teachers and practitioners involved in planning or performing measurements, irrespective of the level of measurement uncertainty and irrespective of the field of application. It is also meant to be a reference for governmental and inter-governmental bodies, trade associations, accreditation bodies, regulators, and professional societies.” [1: Scope].

standpoints that in different periods and fields have been assumed on the nature of scientific and technical knowledge. Hence, the definitions of ‘measurement’ may be considered significant indicators for general issues such as the very possibility of true knowledge, and the relation between experiment and modeling. In a situation in which many traditional distinctions have become blurred (a good example is the idea of fully automatic measurement, that in the past would have been plausibly rejected under the assumption that only human beings are properly able to deal with information), measurement science can maintain its role, instead of dissolving in a myriad of technical sub-disciplines, only by recovering a shared fundamental background. This paper is aimed at presenting and interpreting such multiple definitions and standpoints on the basis of a single conceptual framework allowing to compare them and, finally, to argue in favor of the adoption of what could be called an encompassing *generalized concept of measurement*.

The quest for the definition of measurement is a subject to which prof. Ludwik Finkelstein has given a significant contribution. This paper is written in admired, grateful acknowledgment of his work in measurement science, and in memory of his personality.

## 2. Multiplicity

The scientific, technical, and philosophical literature includes many different definitions of ‘measurement’, thus witnessing the interest for the subject and, at the same time, its complexity. This multiplicity can be interpreted according to several complementary criteria, for example as follows.

<i>Criterion</i>	<i>Exemplary definition</i>
- Is measurement characterized by the structure of the process	“To measure a quantity means to define a unit and to establish how many times the unit is contained in the given quantity. The measurement result is expressed by a number, which expresses how many times the given quantity is greater (or possibly smaller) than the selected unit.” [5] (translated from Italian)
or by the results it produces?	“Measurement is essentially a production process, the product being numbers.” [6]
- Does measurement imply the comparison to a reference, possibly a unit, or not?	“Measurements are executions of planned actions for a qualitative comparison of a measurement quantity with a unit.” [7]  “Measurement is the process of empirical, objective assignment of numbers to the attributes of objects and events of the real world, in such a way as to describe them.” [8]
- Are numbers required products of measurement	“Measurement of magnitudes is, in its most general sense, any method by which a unique and reciprocal correspondence is established between all or some of the magnitudes of a kind and all or some of the numbers, integral, rational, or real, as the case may be.” [9]
or not?	“The only decisive feature of all measurements is symbolic representation; even numbers are in no way the only usable symbols. Measurement permits things (relative to the assumed measuring basis) to be presented conceptually, by means of symbols.” [10]
- Are experimental activities required to perform a measurement	“Measurement is the set of empirical and processing operations performed by means of suitable devices interacting with the measured system with the purpose of assigning a value of a quantity assumed as parameter of the system.” [11] (translated from Italian)
or not?	“Measurement is the assignment of numerals to objects or events according to rule, any rule.” [12]

(the reader is now warned that the concept ‘measurement’ is referred here only to the *process* of measuring and not also to its results). Even more fundamental, because operating at a philosophical level, is the opposition which can be transversally found in these definitions: is measurement a *determination*, i.e., the discovery of an entity existing independently of the measurement, or an *assignment*, i.e., the choice of an entity according to information and criteria derived from the measurement itself? [13]. By supporting either position, measurement scientists and technicians opt, more or less explicitly, for the classical, realist vision of

the world – which “is written in mathematical characters”, as Galileo had synthesized – or for some alternative standpoint, for example under the assumption that “it is *we* who assign numbers to nature. The phenomena themselves exhibit only qualities we observe. Everything numerical [...] is brought in by ourselves when we devise procedures for measurement.” [14].

The incompatibility of these positions could not be expressed more sharply. Of course, this did not prevent pushing forward the leading edge(s) of measurement science, but a confused and confusing situation is troubling its core components.

### 3. An interpretive framework

In order to propose a single conceptual framework, able to present a consistent interpretation for the many available definitions of ‘measurement’, let us take into account two basic questions.

Q1. Are *experimental constraints on the process* relevant for the definition of ‘measurement’, i.e., should the definition include the reference to any experimental conditions?

An affirmative position assumes that only under given conditions on the way the process is performed (typically: operative comparison to a calibrated standard) an evaluation is to be considered a measurement. On the contrary, a negative position considers that experimental constraints are immaterial for characterizing measurement.

Q2. Are *formal constraints on the measured entities* relevant for the definition of ‘measurement’, i.e., should the definition include the reference to any algebraic conditions?

An affirmative position assumes that only if applied to entities fulfilling given formal conditions (typically: empirical ratio leading to a number, as in the Maxwell’s equation,  $\{Q\}=Q/[Q]$ ) an evaluation is to be considered a measurement. On the contrary, a negative position considers that formal constraints are immaterial for characterizing measurement.

Under the generic assumption that measurement is a process aimed at obtaining one or more values that can be attributed to a property of an object (a loose quotation from the VIM3 definition [1: 2.1]):

– Q1 relates to the characterization of measurement as a specific kind of evaluation: if not any evaluation is a measurement, how is measurement specified?

– Q2 relates to the characterization of measurable properties: if not any property is measurable, how are measurable properties specified?

These questions can be interpreted in purely *structural* terms: Q1 refers to the structure of the measurement process, whereas Q2 to the structure of the set of possible measurement results. Hence, the *nature* of the process remains unspecified, and any position about Q1 and Q2 is in principle compatible with all answers to the further question: is measurement applicable only to physical quantities? This agnosticism has the important consequence that a characterization in terms of Q1 and Q2 does not prevent admitting “soft” measurement, and in fact could pave the way to a “widely defined” concept of measurement [4], encompassing the evaluation of both physical and non-physical properties [2]. Of course, further dimensions might be taken into account, but any standpoint on measurement has to account for its position with respect to Q1 and Q2.

It may be finally noted that Q1 and Q2 are independent of one another: the fact that a given position is assumed on one question does not constrain, in principle, a position on the other one. Hence, if for simplicity’s sake both Q1 and Q2 are taken into account as just yes-no questions, i.e., no intermediate, “partially affirmative” positions are allowed (non-binary positions should be in fact admitted, by taking into account the contents of the constraints. For example, relatively to Q2 a distinction could be made by imposing that either only ratio quantities or also ordinal ones are measurable, as assumed, e.g., in the VIM3. On the other hand, this refinement does not modify the arguments that follow and therefore it will not be further developed here), four general positions can be identified:

$\alpha$ : both experimental and algebraic, or

$\beta$ : experimental but not algebraic, or

$\gamma$ : algebraic but not experimental, or

$\delta$ : neither experimental nor algebraic constraints are relevant for the definition of ‘measurement’.

These options can be synthesized as in Figure 1.

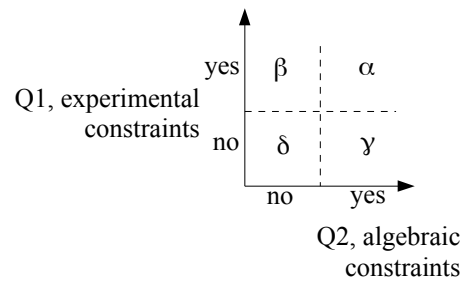


Figure 1 – Options on the definition of ‘measurement’

This option space is in fact a partially ordered set, where each transition from the origin along an axis corresponds to assuming a stronger position on the concept of measurement, where thus  $\delta$  is the weakest position and  $\alpha$  is the most demanding one.

#### 4. A sketch of the historical development of the concept

In the light of the option space  $\alpha$ – $\delta$  a synthetic reconstruction of the historical development of the concept of measurement can be proposed, where each step corresponds to a general answer to Q1 and Q2 and therefore to a position in the option space.

The Euclidean (model of) geometry set the stage first, where the definition is given that “a magnitude is a part of a magnitude, the less of the greater, when it measures the greater; the greater is a multiple of the less when it is measured by the less; a ratio is a sort of relation in respect of size between two magnitudes of the same kind” (Euclid, Elements, Book V, definitions 1–3). Having been formulated under the assumption that the characterization is given of the “real geometry of the world”, no mention of experimental constraints was required for this conception, which is indeed purely algebraic. Hence, this is the original case of the position  $\gamma$ , which may be synthesized as *measurement as quantification*.

The importance of this position is manifest, as quantity calculus / dimensional analysis, and then the International System of Quantities and Units, are founded on it [15, 16]. The very expression “weights and measures” (as if weights were not “measures”, thus evidently assuming that ‘measures’ are to be reserved to geometric entities) is a lexical fossil witnessing the historical centrality of the Euclidean model. On the other hand, such model was devoted to characterize measures, not measurements: as a consequence, the lack of any experimental specifications on the way values are obtained led to a very generic concept of measurement, such that, e.g., “according to my experience, I can see that this object is 1.2 m long” fulfills the required constraint and therefore would be reported as a measurement result.

Centuries after, in the context of the adoption of the experimental method, the Galilean motto of measuring what is measurable and making measurable what is not yet was meant primarily as a call for innovation in instrumentation, an attitude that has been interpreted as a sharp discontinuity with the previous tradition. For example, in reference to the science before Galileo A. Koyré considered that “no one had the idea of counting, of weighing and of measuring. Or, more exactly, no one ever sought to get beyond the practical uses of number, weight, measure in the imprecision of everyday life.” [17]. The Euclidean characterization was maintained, but complemented with the interest in discovering physical transduction effects and introducing devices implementing them. This emphasis on experimental activities was very effective in making measurable quantities that never had been measured before, such as pressure and temperature, and all electrical and magnetic quantities. Once a tool for natural philosophers (i.e., physicists, as they were called at the time of Galileo and Newton), measurement became also a matter for engineers, thus according to the position  $\alpha$ .

As mentioned above, this is the most demanding position, plausibly too strict for the endeavor of extending the Galilean paradigm of making measurable what is not yet in the case of non-physical properties. Measurability was targeted in psychophysics first, since the mid of the XIX century, and then extended to psychology (sometimes under the name of psychometrics) and generally social sciences. With the development and the widespread adoption of measurement-like tools, like the well known Stanford-Binet test for evaluating the so-called Intelligence Quotient, the issue was becoming so critical that in the '1930 the British Association for the Advancement of Science appointed a Committee, jointly composed of physicists and psychologists, to study the possibility of providing “quantitative estimates of sensory events”, as the final report, issued in 1939, stated. No commonly agreed position was reached, as the physicists maintained the traditional position on measurement [18], thus arguing against the possibility for non-physical properties to be properly measurable [19]. This triggered a thorough activity, mainly made by philosophers and social

scientists, aimed at rethinking the foundational concepts around measurement, abstractly thought of as a representation process. And since even a rough characterization such as “measurement is the process performed by a physical measuring system” does not apply in this case, the formal structure of property sets and the possibility of their representation by means of symbolic entities became the main topic of analysis. From the seminal paper by H. von Helmholtz on “counting and measuring” [20] a theory of (so called) measurement scales was proposed by S. S. Stevens [21], then developed into an axiomatized representational theory of measurement [22]. Here no trace remains of experimental constraints, and the theory may be interpreted in fact as relating to scale construction, not measurement as such [23, 24]. The generalization obtained on algebraic constraints, as modulated by the Stevens’ theory of types, led to a standpoint effectively expressed by the already quoted statement: “measurement is the assignment of numerals to objects or events according to rule, any rule” [12]. Hence, this appears to be *a position  $\delta$* .

This generality explains why the representational theories are seldom used in physics and engineering (as an example, they are not even mentioned in the VIM3; the diagram of Figure 1 suggests that such theories are in a sense at the opposite side with respect to the tradition of physical measurement), despite the remarkable effort particularly devoted by L. Finkelstein to this purpose since the early stages of the development of such theories (e.g., [25, 26]). On the other hand, even in the social sciences a so general position generated several objections. For example, in the context of Rasch measurement, typically used in in psychometrics [27], it is considered that ordinal properties are not specific enough to be considered measurable, and the search for at least interval scale evaluations becomes a critical issue. Even though the lack of a widespread metrological infrastructure, such as the one founded on the International System of Units, hinders in this case the adoption of primary measurement standards, this seems to be a process towards the Euclidean interpretation, i.e., *the position  $\gamma$* .

In the meantime, physical sciences and technology stuck in the traditional standpoint, as witnessed by the three editions of the VIM. As for experimental constraints, Q1, the first two editions [28, 29] defined ‘measurement’, rather implicitly, as “set of operations having the object of determining the value of a quantity” (“a value of a quantity”, in the VIM2). A clearer position has been taken by the VIM3, that proposes the definition “process of experimentally obtaining one or more quantity values that can reasonably be attributed to a quantity” [1: 2.1] and then among the “presupposed” conditions lists “a calibrated measuring system operating according to the specified measurement procedure, including the measurement conditions” [1: 2.1 Note 3]. With respect to algebraic constraints, Q2, the concept ‘(measurable) quantity’ has been redefined: while in the first two editions quantities were defined as properties with a measurement unit, i.e., properties in “ratio scale” in the Stevens’ terminology, in the VIM3 the scope of measurement has been extended also to ordinal properties. On the other hand “measurement does not apply to nominal properties” [1: 2.1 Note 1], i.e., an algebraic constraint is however maintained, thus assuming a standpoint halfway *between the positions  $\gamma$  and  $\alpha$* .

Hence, according to this sketchy reconstruction all positions have been historically explored in the option space  $\alpha$ - $\delta$ , but  $\beta$  (see Figure 2).

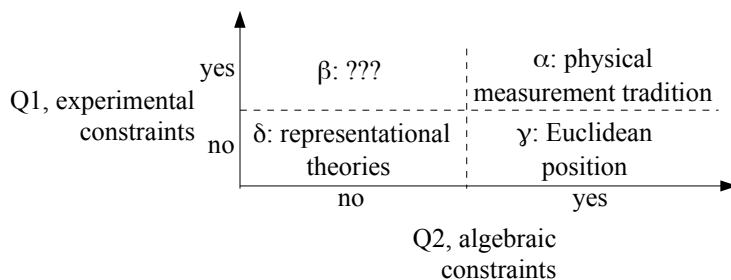


Figure 2 – Options on the definition of ‘measurement’, as historically proposed.

It is exactly the position  $\beta$  that we aim at exploring here.

### 5. Towards a generalized concept of measurement

Let us reconsider the two general issues introduced in Section 3:

- how is measurement characterized as a specific kind of evaluation?
- how are measurable properties characterized as specific kinds of properties?

As we have argued, along the traces of the Euclidean tradition the second question has been much more emphasized than the first one, up to a point that measurement has not been distinguished from quantification

(“it is a widespread belief that [...] (i) quantitative properties appear upon measurement and (ii) measurement generates quantitative concepts” [30]). That the property evaluation type, i.e., the algebraic structure which is preserved in the representation [31], affects the conveyed structural information is not under discussion, so that, e.g., a ratio evaluation is obviously more informative, *ceteris paribus*, than an ordinal one [32]. On the other hand, a constraint such as, e.g., “measurement applies to at-least-ordinal properties” (the position of the VIM3, as mentioned) if taken as definition, i.e., measurement is an evaluation that applies to at-least-ordinal properties”, appears unable to justify the reliability which is customarily attributed to and expected for measurement results. According to the position  $\beta$ , such reliability has to be looked for in the experimental structure of the process, not in its algebraic type. Furthermore, the specified structure should be characterized in terms of the functional, instead of physical, features of the process (even though such physical features can be exploited to classify measuring instruments). This keeps into account the manifold variety of measuring systems and at the same time opens the doors to applying the characterization also to the evaluation of non-physical properties.

Measurement is a designed process, planned and performed on purpose. Hence, in the quest for its definition a pragmatic standpoint may be appropriately adopted, by focusing on the epistemic features expected for measurement results. Our proposal is that for an evaluation to be considered a measurement its results must convey [33]:

- information specific to the measurand and independent of any other property of the object or the surrounding environment, including the subject who is measuring: this is a requirement of *objectivity*, i.e., relatedness to the object;
- information interpretable in the same way by different users in different places and times: this is a requirement of *inter-subjectivity*, i.e., unambiguous representation.

As characterized here, objectivity and inter-subjectivity are mutually independent features – an evaluation might produce objective but non inter-subjective results, or inter-subjective but non objective results – so that both these conditions are required.

According to an idealized model, a physical measuring instrument is the prototype of a system able to produce objective and inter-subjective information on the measurand, since:

- it behaves as a transducer which is able to filter out all effects due to influence properties and is sensitive only to the property subject to measurement, so that its output provides information specific to the measurand, and therefore objective results;
- it behaves as a transducer which is calibrated and is able to indefinitely maintain its calibration state, so that its output provides information stably traceable to a primary measurement standard, and therefore inter-subjective results.

This description complies with the mentioned condition given by the VIM3 on measurement, a process performed by *a calibrated measuring system operating according to a specified measurement procedure* [1: 2.1 Note 3]. Moreover, the fact that no measuring instruments perfectly fulfill these conditions is the basic justification why instrumental measurement uncertainty must be generally included in the uncertainty budget, and measurement uncertainty has to be compared to target measurement uncertainty [34] “on the basis of the intended use of measurement results” [1: 2.34], i.e., for pragmatic reasons, typically supporting decision making processes.

Such concepts of objectivity and inter-subjectivity are not specifically bounded to the physical nature of the measuring instruments. Rather, they can be assumed as requirements towards a *generalized concept of measurement*, applicable to both physical and non-physical properties, and therefore generalizing the position  $\alpha$  to the position  $\beta$ . The abstract functional structure of measurement that may be derived is depicted in Figure 3.

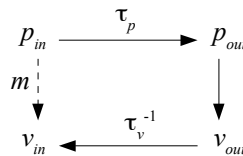


Figure 3 – The structure of measurement.

The measurement of the property  $p_{in}$  produces the property value  $v_{in}$ , i.e.,  $p_{in}$  is represented by the value  $v_{in}$  through a representation function  $m$  (measurement uncertainty is neglected here), because:

1. the property  $p_{in}$  is experimentally transduced to a property  $p_{out}$  by means of a device whose behavior is a realization of the transduction function  $\tau_p$ ;
2. the transducer output property  $p_{out}$ , i.e., the instrument indication, is mapped to an indication value  $v_{out}$ ;

measuring instruments are designed so that this mapping is performed in unproblematic way, as, e.g., the observation of coincidence of marks on a scale, the classification of an electric quantity to a quantized level to which a digital code is associated, the numbering of right answers of a test, ...;

3. an informational version of the transduction function  $\tau_p$  is available,  $\tau_v$ , which maps input property values  $v_{in}$  to indication values  $v_{out}$ ; furthermore, such function is assumed to be invertible, so that the indication value  $v_{out}$  is mapped to the measurand value  $v_{in}$  through  $\tau_v^{-1}$ .

Of course, providing the function  $\tau_v$  is the purpose of the transducer calibration, whose functional structure is depicted in Figure 4.

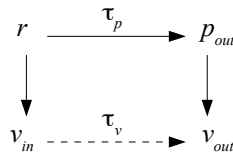


Figure 4 – The structure of calibration.

In this case:

1. as for measurement the transducer is operated and an indication is obtained; in this case the value  $v_{in}$  for the input property is assumed to be previously known via a traceability chain, being  $r$  a reference property typically realized by a measurement standard;

2. as for measurement the instrument indication  $p_{out}$  is mapped to an indication value  $v_{out}$ ;

3. the calibration function  $\tau_v$  results from a set of couples  $(v_{in}, v_{out})$  together with some additional hypothesis (e.g., linearity, polynomial interpolation, ...).

This description meets the expectations introduced above:

– it accounts for the claimed objectivity and inter-subjectivity expected for measurement results;

– it applies to the evaluation of both physical and non-physical properties,

while

– it does not introduce any constraints as for the measurable properties,

so that the whole range of types of evaluations, from counting (“absolute properties”) to classification (“nominal properties”) can be taken into account as candidate to measurement.

Hence this standpoint conforms to the position  $\beta$ : measurement is characterized as a property evaluation whose experimental structure is able to produce values having a sufficient degree of objectivity and inter-subjectivity with respect to their intended use. Measurement is uncorrelated with quantification: the measurability of a property is a feature derived from experiment, not algebraic constraints.

## 6. Conclusions

In one of his last scientific paper on measurement [35], L. Finkelstein argued that “there are a range of problems of widely-defined measurement that require addressing. They constitute a research agenda. Among them are the need to engage in the history and philosophy of science and the methodology of the sciences in which measurement is applied.” Among the fundamental “properties of measurement arising from the wide-sense definition” of the concept, he acknowledged that “measurement provides an objective description of the measurand. The description is not merely a matter of opinion or feeling. It is invariant in rational discourse. [...] Given a specification of the measurement process the same symbolic description of a measurand should in principle be obtainable by any observer.”

Restating these properties in terms of objectivity and inter-subjectivity of measurement results, the present paper has been aimed at providing a contribution to bridge currently different conceptions of ‘measurement’ and reaching a socially-responsible, unified, encompassing concept: a task also aimed at supporting the blossom of Ludwik Finkelstein’s heritage.

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