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Paper for TIM – special issue on Measurement Fundamentals

MEASUREMENT, MODELS, UNCERTAINTY

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Abstract
Against the tradition, which has considered measurement able to produce pure data on physical systems, the unavoidable role played by the modeling activity in measurement is increasingly acknowledged, in particular with respect to the evaluation of measurement uncertainty. The paper characterizes measurement as a knowledge-based process, and proposes a framework to understand the function of models in measurement and to systematically analyze their influence in the production of measurement results and their interpretation. To this aim, a general model *of* measurement is sketched, which gives the context to highlight the unavoidable, although sometimes implicit, presence of models *in* measurement and finally to propose some remarks on the relations between models and measurement uncertainty, complementarily classified as due to the idealization implied in the models and their realization in the experimental setup.

1. Introduction
It is nowadays customarily admitted that models are in principle necessary tools to design measurement processes, represent their results, and interpret the information they provide, particularly in assessing the quality of such information as expressed by measurement uncertainty. On the other hand, modeling activities are often studied and developed as mainly concerning the definition of mathematical procedures for analyzing and processing data, as in the significant case of uncertainty propagation, thus focusing these activities on the informational, instead of the experimental, stages of the measurement process. In the scientific literature it is indeed not frequent to find analyses of the way models contribute to the whole picture, by identifying the specific stages of where models are plugged in and the role they play in introducing or increasing uncertainties (a noteworthy exception is, e.g., [1] and [2]). Despite the epistemological connotation of the topic, some contributions may be obtained by this analysis to the measurement practice by making the structure of the modeling activity explicit and highlighting the significant points where models are implicitly at work.

This paper proposes an introduction to the subject: on the basis of a simplified model of the measurement process, the elements of the process that are subject to the modeling activity are identified and classified. The outcome of the analysis is twofold: first, the modeling activity reveals to be pervasive and influencing every stage of the measurement process; second, the identification of the steps of such activity turns out to be a useful tool to assess and classify the sources of measurement uncertainty. The paper is organized as follows. In Section 2 an overview of the conceptual framework is provided, in which the problem of the connections between measurement, models, and uncertainty is settled, also in a diachronic perspective. In particular, two significant changes are underlined: (i) the move, in epistemology, from pure to modeled data, and (ii) the move, in metrology, from an error-dependent conception of uncertainty to a model-dependent (i.e., information-oriented, knowledge-based) one. In Section 3 a general model *of* measurement is sketched, which gives in Section 4 the context for discussing the function of models *in* measurement. On this basis, in Section 5 we propose some considerations on the relations between modeling and measurement uncertainty, thus emphasizing in particular how models are sources of uncertainties.

2. The emergence of models
Measurement has been traditionally considered a process able to produce *pure data*, i.e., data independent of any interpretation, on physical systems. The basic idea underlying such conception is that physical systems are characterized by physical quantities whose magnitudes do exist in themselves, being inherent features of the systems. Aim of measurement is then to provide information about such magnitudes in the form of quantity values. However, due to the complexity of the interaction with the physical world, measuring systems acquire only approximate information on physical systems and the values they provide must be considered as estimates of *true quantity values*, thought of as the values produced by ideal measuring systems operating in the best measurement conditions, when all measurement errors have been removed. In this sense, “the uncer-

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tainty of the result of a measurement reflects the lack of exact knowledge of the value of the measurand" [3], and as such it derives from the incompleteness of the available information concerning (i) the context in which measurement is performed, including the quantities influencing the object under measurement and the measuring system, and (ii) the accuracy (i.e., a superposition of precision and trueness), of the measuring system [4]. Were such incompleteness removed, the measurand value would be completely known. The traditional concept of ideal measurement is then characterized as a process performed by means of a perfectly accurate measuring system, operated according to a completely specified procedure, and coupled with the object under measurement so to constitute a system which is closed or however perfectly known in its behavior.
A deep revision of this conception resulted from a general reflection about history and sociology of science, not from measurement science. Since about fifty years, philosophers are highlighting that pure data do not exist [5-10], as the information about the physical world is both acquired and interpreted in the light of models and hypotheses. It is not uncommon today to read statements such as "observations are never interpreted independently of some abstract model of the physical system, analyses and calculations of results are done on the model quantities which are supposed to correspond to the physical quantities." [9, p.4]. The consideration of the role of models in science was the novelty, which eventually has reached measurement science.
Such change is relatively recent, and is well witnessed by the shift in the meaning defined for the concept 'measurement uncertainty' in the *International Vocabulary of Metrology*, the so called VIM. In its first edition, 1984, measurement uncertainty was defined as "an estimate characterizing the range of values within which the true value of a measurand lies" [11]. Less than twenty-five years after, in the current edition, 2008 (VIM3), the definition has become "non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used" [12]. The lexical choices are significant here: *quantity values are not determined but attributed*, i.e., they are not assumed as inherently tied to the measurand anymore, and such attribution is not based on truth (of values) but on information (on values) [13].
The acknowledgment that uncertainty is part of measurement, and that through uncertainty the assumed quality of measurement results is stated, presents measurement as a knowledge-based process, where uncertainty evaluation is a pivotal task, with a theoretical background [14] but critical operative implications: "the most meaningful issue of a measurement practice is the estimation of uncertainty that must be associated with the result of a measurement so that it can usefully be employed in any technical, commercial, or legal activity." [15]. Under the hypothesis that the measurand Y depends on $n \geq 1$ quantities X_i via a function f, a situation sometimes called of indirect measurement, such task is customarily coped with by assuming that the information on the "input quantities" X_i is available as a vector $m = (m_1, ..., m_n)$ of measured quantity values and their $n \times n$ covariance matrix $M = (u_{i,j})$. Hence, the problem is to propagate this information through f so to obtain the required information on Y. The basic solution assumes approximating f by linearization, i.e., as a first-order Taylor series, so to make it possible to operate separately on the measured quantity values and their uncertainties, by computing $m_Y = f(m_1, ..., m_n)$ and estimating its standard uncertainty u_Y as a function f of M [3]. Despite the advantages due to this simplicity, and its connection with the traditional "error propagation", this technique has some known drawbacks, mainly related to the approximation requirement itself and the need to obtain the first partial derivatives of f, thus immediately implying its inapplicability whenever the function has an analytical complex form, e.g., as an algorithm. The current scientific literature witnesses the several research activities that have been devoted to generalize this framework and to overcome such drawbacks, in particular for exploring alternative methods to compute the measured quantity value m_Y when f is significantly nonlinear [16], computing m_Y and u_Y via numerical techniques that do not require the linearization of f and avoid computing its partial derivatives, such as the unscented transform [15], generalizing the concept of measurement model and its formal treatment in the multivariate case, i.e., to vector measurands [17]. The hypothesis of formalizing the available information on the input quantities in terms of probability distributions, and not only some of their moments, has been supported, and the numerical propagation of such distributions by means of Monte Carlo methods analyzed [18, 19]. Precisely the acknowledged relevance of models has triggered the consideration of interpreting probabilities in measurement according to a Bayesian approach [20, 21], as an alternative or complement to their traditional statistical understanding. Furthermore, non-probabilistic frameworks have been proposed to formalize the problem, particularly in the context of the mathematical theory of evidence [22], typically by means of the fuzzy set theory [23, 24] or the theory of random-fuzzy variables [25].
As it can be seen, in this context models are primarily conceived as mathematical tools aimed at expressing and combining uncertainties to produce measurement results that are as informative and reliable as possible (significantly, the "mathematical relation among all quantities known to be involved in a measurement", "a general form" of which "is the equation $h(Y, X_1, ..., X_n) = 0$", is called *measurement model* in [12]). On the

other hand, much less explored has been the whole framework, where the function of models in measurement is systematically analyzed and their influence in the production of measurement results and their interpretation is specified. In this perspective, we propose to consider measurement results as obtained from a process which (explicitly or implicitly) includes a modeling activity, about *both* the measurement process and the measurand, where:

- *a model about the measurement process* specifies the quantities involved in the process, the kinds of their interactions, and the conditions of such interactions;
- *a model about the measurand* specifies the relevant structure of the object under measurement, the definition of the measurand itself in reference conditions (i.e., as if it were measurable by an ideal measuring system), and the set of quantity values which could be assigned to the measurand [26].

The basic thesis of this paper is that measurement uncertainty crucially depends on both such models. As obtained by a measurement process, the knowledge on the measurand is incomplete primarily because of (i) the idealization of the object under measurement, and (ii) the realization of the ideal measuring system, as assumed / defined before measurement in a modeling activity.

3. The sketch of a model of a measurement process

According to its most general black box model, measurement is described as an empirical, fundamental process producing an entity, called the *measurement result*, which is supposed to convey information on a given entity, let us call it the *object under measurement*. More specifically, what is measured is a property (e.g., a physical quantity) characterizing the object under measurement, i.e., the *measurand*, and the information entity is produced by properly representing the outcome of a physical interaction between the object under measurement and a measuring instrument in a specific environment. It is thus crucial acknowledging and maintaining the distinction between:

- the physical outcome of the interaction, typically the output signal provided by a transducer (let us call it "output quantity" for short), and
- the informational outcome of the representation, i.e., the quantity value that, "together with any other available relevant information" [12], conveys information on the measurand.

Accordingly, a measurement process can be decomposed into two stages:

- an *empirical* stage, where the interaction between the object and the instrument is accomplished and an output quantity is obtained;
- an *informational* stage, where the output quantity is represented as a measurand value.

In the simplest case, when coupled with the object under measurement the measuring instrument acts (i) as a *selector*, interacting with the object under measurement with respect to a given quantity, the measurand, and (ii) as a *comparator*, interacting in such a way to produce a comparison with a set of measurement standards (in [27] it is suggested that the capability of measurand selection and comparison to standards are the functional justification of the claim that measurement results convey *objective* and *inter-subjective* information respectively). Both the selection and the comparison are usually imperfectly implemented in the measurement process, in particular because: (i) measuring instruments might be sensitive not only to the measurand but also to *influence quantities* (of the object under measurement itself, the measuring instrument, the environment), under the acknowledgment that the coupled system is not perfectly closed with respect to the environment; (ii) the comparison between the object under measurement and the measurement standards is usually performed asynchronously, where the measuring system operates as a memory unit and it might not be perfectly stable in this function. In order to better highlight the basic characteristics of a measurement process, in the rest of this Section a twofold idealization will be introduced, assuming both perfect closure and perfect stability. The discussion of these issues is left to the next Section.

A quantity value deriving from such empirical + informational process is structurally obtained in three steps.

Step 1: mapping from quantities subject to measurement to output quantities.

The core component of a measuring system is a device which operates by transducing the quantity being measured, Q_{in}, to an output quantity, Q_{out}, where such behavior is formalized by a *transduction function* τ_Q, as shown in Fig.1.

$$Q_{in} \xrightarrow{\tau_Q} Q_{out}$$

Figure 1 – Basic transduction.

Although measuring instruments can be chained, so that the output quantity of a transducer is the input quantity of another transducer, the output of the last measuring instrument of the chain (the only one, usually) is what results from the empirical side of the measurement process. As a consequence, the sensitivity of the instrument and its resolution determine the conditions how the measuring interval is mapped to the set of output quantities.

Step 2: mapping from output quantities to output quantity values.

Transducers exploited in measuring instruments are designed so that the output quantities they produce (i.e., the output quantities of the last transducer in the chain) are assumed as *primitively discernible*, in some experimental sense, depending on the specific nature of the output quantities themselves. This corresponds assuming that each output quantity can be mapped to an output quantity value, as shown in Fig.2.

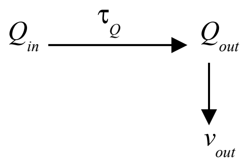

Figure 2 – Transduction followed by representation by means of transducer output quantity values.

This mapping can be abstractly characterized according to the following conditions:

- specification of a classification, implying the definition of a procedure to recognize the membership of any given quantity to a class;
- assignment of labels to each class, so to represent classes by means of output quantity values;
- recognition of the membership of the actual output quantity to a given class, so that the corresponding label is assigned.

Step 3: mapping from output quantity values to measurand values.

As shown in Fig.3, the mapping of output quantity values to measurand values is finally obtained by the instrument calibration, which has to elicit the information on the transduction behavior and to guarantee the traceability of the measurement results to a given reference.

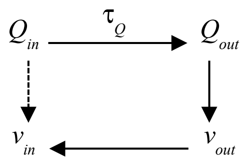

Figure 3 – Mapping of transducer output quantity values to measurand values.

Indeed, as shown in Fig.4 for any given *reference quantity* S, as typically realized by a measurement standard and such that s is the quantity value assigned to it in the traceability chain, instrument calibration is aimed at producing a map $v_{out} = \tau_v(s)$, and such that an appropriate set of references, $S = \{S_i\}$ is available and each S_i is mapped to a different quantity value s_i (in the notations τ_Q and τ_v the subscripts aim at highlighting the fundamental difference in the context of measurement between *quantity* equations and *quantity value* equations, respectively describing experimental and informational facts).

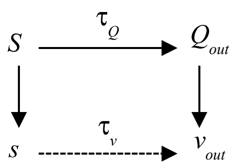

Figure 4 – Calibration.

Measurement exploits such information by inverting it, $v_{in} = \tau_v^{-1}(v_{out})$, as in Fig.5, so that the output quantity value v_{out} is associated with a measurand value v_{in} which, because of the hypothesis of transducer stability, is assumed to be traceable to the reference S.

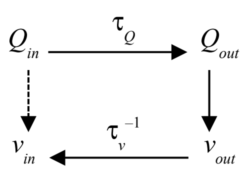

Figure 5 – Measurement.

From the fact that, in the case of measurement of physical quantities, this behavior is typically based on a physical effect, three categories of calibration conditions can be considered:

A. *process based on non-parametric laws:* the transducer behavior perfectly realizes a non-parametric physical law, where all effects of influence quantities are included in the law; in this (clearly ideal) case, no calibration would be necessary: once the transducer has been initially characterized, the law is sufficient to assign measurand values;
B. *process based on parametric laws:* the transducer behavior realizes a parametric physical law, where some of the parameters might keep into account undetermined or underdetermined effects due to influence quantities; in this case calibration has to be performed in order to remove all the degrees of freedom in the law, and it exploits the structure of the law; for example, if a linear dependence is assumed, then two distinct references are sufficient;
C. *process based on the hypothesis of causality only:* the transducer behavior realizes a transformation which is analytically unknown, and only a black box model is given that assumes a causal behavior, i.e., the transducer output is determined by the transducer input; in this case calibration has to be performed and requires multiply probing the transducer to generate an appropriate sequence of (input quantity value, output quantity value) data, thus typically in the form of a lookup table.

Hence, all three categories imply a model of the measuring instrument behavior, including its dependence on influence quantities, from the strongest model (conditions A), which requires a complete knowledge of the modeled system, to the weakest one (conditions C), where only its causal behavior is supposed. For all three categories a further general constraint is assumed. Since τ_v^{-1} is applied in measurement as the information-related inverse of the mapping $Q_{in} \rightarrow Q_{out}$, performed at the time t_m, whereas it was obtained in calibration by $S \rightarrow Q_{out}$, performed at the time t_c, $t_m \neq t_c$, the hypothesis is that the transduction function τ_Q did not change in the interval $[t_m, t_c]$. Accordingly, the model assumes in particular the *stability* of the measuring instrument. On the other hand, the structure of the process depicted in Fig.5 emphasizes the complementary role of the two stages of which measurement is composed: (i) in the empirical stage the output quantity is experimentally *determined*, but is still *unknown*; (ii) at the end of the informational stage, the output quantity turns out to be *known*, but the outcome, i.e., the measurement result, is only obtained through a model of the measurement, and therefore it should be considered as *assigned* [13]. Hence, this simple model *of* the measurement process highlights the interest of studying the role of models *in* measurement. As it will be discussed, the significance of the information obtained in measurement critically depends on the models exploited in both the empirical and the informational stage of measurement.

4. Highlighting the role of models in measurement

Any measurement is performed with a goal which, explicitly or implicitly, gives the specifications of the process and whose achievement justifies the resources to be employed: hence, any measurement is aimed at solving a *measurement problem*, stated in terms of the object under measurement and the measurand, and interpreted in view of the measurement goal. With a simple example the role of models in a measurement problem can be presented. Let us suppose that the electrical resistance R of a given component c has to be measured for some goal, so that the measurement problem is specified in particular by the pair (c, R). A solution to the problem implies accomplishing three activities.

Activity 1: *analysis - idealization*. A twofold idealization is typically performed: first, a model of the object under measurement is introduced, e.g., the component c is modeled as a resistor c^* obeying the Ohm's law; second and consequently, the measurand is defined, e.g., as the electrical resistance R^*, evaluated using positive real numbers and not influenced by any physical quantity, such as temperature. In this way the measurement problem:

(c, R)

becomes the ideal measurement problem:

(c^*, R^*)

Of course, such idealization is not imposed to the modeler, who by means of these models actually decides the concepts (of the object under measurement and the measurand) she considers appropriate in dependence of her goals. For example, a general option to reduce the requirements as for the measurand idealization is to include in its definition all the environmental conditions and to maintain them implicit ("the measurand is the electrical resistance of the component in this moment in these conditions, unknown and whatever they are"). This position has the operative immediate benefit of avoiding the need to measure any influence quantity, but it prevents projecting the acquired information outside the strict (and unknown) context in which it has been obtained.

Activity 2: *ideal solution*. A solution to the identified ideal problem is then searched, also by decomposing it into a set of more tractable problems if appropriate, e.g., by assuming that by means of the Ohm's law a quantity value for R^* of c^* can be obtained by simultaneously measuring the electrical potential difference V^* and the current intensity I^* across c^*. The ideal problem (c^*, R^*) is thus decomposed into two ideal problems:

(c^*, V^*)
(c^*, I^*)

assumed to be solvable by ideal measurement systems, e.g., an ideal voltmeter m_V^* and an ideal ammeter m_I^*, together with an ideal measurement procedure specifying how m_V^* and m_I^* are to be applied to c^* to find the quantity values of V^* and I^*, where the hypothesis of ideality for m_V^* and m_I^* supposes their results to be perfectly traceable to appropriate references. As a consequence, the solution to the ideal problem has the following structure:

- apply m_V^* and m_I^* to c^* according to the procedure and find a quantity value v^* of V^* and a quantity value i^* of I^*;
- divide v^* by i^* and find the result, i.e., a quantity value r^* of R^*.

In this activity, both empirical and mathematical operations are specified.
Activity 3: *synthesis - realization*. Once a solution to the ideal problem has been found, a twofold de-idealization is typically required: first, to obtain some instances m_V and m_I of the ideal measurement instruments m_V^* and m_I^*; second, to couple the object under measurement with the measuring instrument(s). In this way, a solution of the measurement problem is found according to the procedure specified in the ideal solution. As a consequence, the actual solution has the following structure:

- obtain m_V and m_I;
- apply m_V and m_I to c according to the procedure and find a quantity value v of V and a quantity value i of I;
- divide v by i and find the result, r, interpreted as a quantity value of R.

The diagram in Fig.6 depicts the structure of the process.

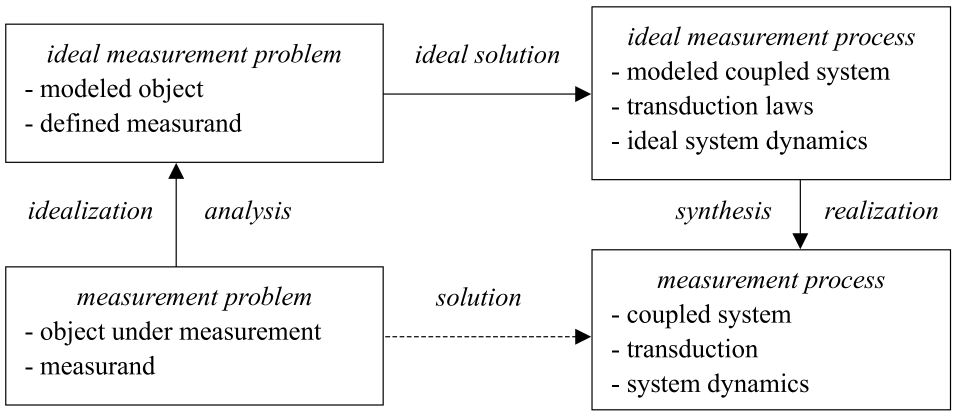

Figure 6 – Structure of solution of a measurement problem.

This structure highlights that the validity of the obtained results rests not only on the empirical quality of the measurement, as traditionally acknowledged, but also, and critically, on (i) the adequacy of the idealization of the measurement problem and (ii) the degree of correspondence between the ideal and the actual measurement process. In the example, the measurement problem, as presented, is idealized for several, well known, reasons, in particular the assumptions of lumped parameters and of independence from temperature (i.e., in the relation:

$$R(T) = R_0 \, [1 + \alpha \, (T - T_0)]$$

the parameter α is hypothesized to be null; even in the refined case where $\alpha \neq 0$, an idealization would be assumed, in the hypotheses that α is a constant and that the dependence of R on temperature is linear); in its turn, the realization of the ideal measurement process implies some assumptions, in particular about the actual contemporariness of the application of m_V and m_I, and the choice between the alternative options shown in Fig.7, where either the internal resistance of the ammeter is assumed negligible (left diagram) or the internal resistance of the voltmeter is assumed very large (right diagram).

Figure 7 – Two alternative options to realize the ideal solution of an exemplary measurement problem.

Of course, enhancing the quality of the involved models, i.e., reducing their "distance" to the actual systems and processes, is possible, but at the cost of increasing the complexity of the whole measurement process and the resources required for its execution (e.g., if the dependence on temperature of resistance is included in the model, then the process must be modified to include the measurement of temperature). Hence, both (i) and (ii) imply a model-related decision on the trade-off between model accuracy and resources to be employed, and therefore between quality of measurement results and costs of the measurement process, where the measurement goal gives the decision criterion.

5. Accounting for uncertainty in measurement processes

The previous considerations give an encompassing context to discuss about the role of uncertainty in measurement. This will be done in two steps, by first analyzing how uncertainty is accounted for in the *Guide to the expression of uncertainty in measurement* (GUM) [3] and how such account can be incorporated in a system theoretical analysis, as proposed in [1, 2], and then by suggesting how this analysis can be extended in future developments.

According to the GUM (paragraph 3.3.2) different possible sources of measurement uncertainty are acknowledged, roughly classifiable into the following groups.

Group I, concerning the *measurand*, e.g., incomplete definition of the measurand; imperfect realization of the definition of the measurand.

Group II, concerning the *measurement procedure*, e.g., assumptions incorporated in the measurement method and procedure; approximations incorporated in the measurement method and procedure.

Group III, concerning the *sources*, e.g., inexact values of measurement standards and reference materials; inexact values of constants and other parameters obtained from external sources.

Group IV: concerning the *environment*, e.g., imperfect measurement of environmental conditions; imperfect knowledge of the effects of environmental conditions.

Group V, concerning *instruments*, e.g., bias in reading analogue instruments; finite instrument resolution or discrimination threshold.

An interpretation of how uncertainty originates can be obtained by analyzing the equation which describes the dependence of the measurand on the quantities that are involved in the measurement process. Two general cases should be taken into account, whether (α) the transducer is supposed to be directly sensitive to the measurand (as when potential difference is measured by a voltmeter), or (β) to obtain a measurand value a computation is required (as in the previous example where resistance is computed by potential difference and current intensity). Since in case β the measurement problem is decomposed into one or more α problems and then a computation stage, let us focus here on the α case. The following equation is introduced:

$$Y = f(X, \boldsymbol{Z}) \qquad (1)$$

where Y is the measurand, X is the transducer output quantity and $\boldsymbol{Z}$ a vector of influence quantities, whose values and uncertainties are assigned in the current measurement or by external sources. Eq. (1) can be called the *model equation*, as it represents the connection between the measurand and the quantities which are supposed to uniquely determine it in the given measurement context. Thus, a model equation can be viewed as an inverted version of:

$$X = g(Y, \boldsymbol{Z}) \qquad (2)$$

which describes X as the effect of Y given $\boldsymbol{Z}$. Eq. (2) is a generalized version of the transduction equation introduced above (see Fig.1) and represents the model of a causal connection such that the measurand, Y, is interpreted as one of the causes producing the output quantity, X, in a context characterized by the vector $\boldsymbol{Z}$ of influence quantities (it could be noted that terminology on this matter is unfortunately still far from a general agreement; the VIM3 calls f the "measurement function" [12]; particularly in the study of uncertainty / distribution propagation, see, e.g., [28] and [29], eq. (2) is termed the "observation equation" and eq. (1) the "measurement equation").

This provides the following generic description of the steps leading to measurement:

1) identify Y, X, and $\boldsymbol{Z}$;

2) identify the interaction between Y, X, and $\boldsymbol{Z}$, and model them in terms of the generalized transduction equation, eq. (2);
3) introduce the imperfections due to incomplete knowledge concerning steps 1) and 2);
4) invert the mathematical relation to obtain the model equation, eq. (1), from the generalized transduction equation.
These steps are systematically considered in the description of how uncertainty enters the measurement process proposed by Sommer and Siebert [1, 2]. According to his description, the evaluation of measurement uncertainty is based both on the way the measurement process is construed and on the way such a process is affected by influence quantities. Hence, the evaluation of measurement uncertainty involves the knowledge of both how the process is structured, as represented by the generalized transduction equation, and the role of those quantities, represented by means of probability density functions. In this context three components of measurement uncertainty are acknowledged, related to:
(1) *source units*, representing measurable quantities provided by a source, associated with uncertainties due to possible differences between nominal and actual values (sources in Group III);
(2) *transmission units*, representing signal processing, associated with uncertainties due to possible interferences caused by influence quantities (sources in Group IV);
(3) *indicating units*, representing (transduction) output quantities, associated with uncertainties due to possible limits of resolution (sources in Group V).
Hence, this characterization accounts for the possible uncertainties due to (1) differences between nominal and actual values of standards, (2) the role played by the influence quantities during the process, and (3) the interpretation of the output quantities, all of them taken into consideration in probabilistic terms. On the other hand, important sources of uncertainty, such as those related to the definition of the object under measurement, the definition of the measurand, the definition of the coupled system, the definition of the transduction laws, are out of the scope of this description. Our claim is that *the existence of uncertainties is critically dependent also on the modeling activity concerning the measurand and the measurement process*, so that uncertainty sources in Groups I and II turn out to be significant. In reference to the Activities introduced in Section 4, measurement uncertainty is connected with modeling because:

(i) in the *analysis* step, the introduction of models of entities and quantities involves idealization, and therefore some distortion of how such entities and quantities actually are;
(ii) in the *ideal solution* step, the introduction of models of coupled systems involves the specification of transduction laws linking quantities of different kinds, and therefore influence quantities and interaction laws enter the picture;
(iii) in the *synthesis* step, the realization of entities determined with respect to certain quantities, the standards, involves production, and therefore some distortion of how the entities to be realized ideally are.

By focusing in particular on the uncertainties depending on the specification of the measurand and of the measurement procedure, let us review the example introduced in Section 4, under the hypothesis that the empirical stage of measurement is implemented by transducers mapping potential differences and current intensities to angular positions (as indicated in the display of a voltmeter and an ammeter), whose quantity values are assumed to be known. Then in the informational stage of measurement a value for the measurand, resistance, is obtained by exploiting Ohm's law. Let us denote by $Q_{V,\text{out}}$ the angular position of the voltmeter pointer and by $Q_{I,\text{out}}$ the angular position of the ammeter pointer. The model equation can be now flexibly used:
1) for modeling Ohm's law: $R = f(V, I) = V / I$
2) for modeling the first transduction process, e.g.: $V = f_1(Q_{V,\text{out}}) = \alpha \cdot Q_{V,\text{out}}$
3) for modeling the second transduction process, e.g.: $I = f_2(Q_{I,\text{out}}) = \alpha' \cdot Q_{I,\text{out}}$
4) for modeling the composed function: $R = f_3(Q_{V,\text{out}}, Q_{I,\text{out}}) = \alpha \cdot Q_{V,\text{out}} / \alpha' \cdot Q_{I,\text{out}}$
There may be uncertainties linked both to the transduction processes and to the output quantities, thus interpretable by Sommer's *transmission units* and *indicating units* respectively. But it should be clear now that, in order to characterize such uncertainties, it is crucial to take into account the way in which the measurand and the whole process have been modeled, in particular by making the hypotheses used in defining such models explicit, e.g., the hypotheses concerning:

- the validity of Ohm's law;
- the linearity of the relation between V and $Q_{V,\text{out}}$;
- the linearity of the relation between I and $Q_{I,\text{out}}$;
- the closure of the whole system, and therefore the independence of the mentioned quantities from in-

fluence quantities.

This further supports the thesis that models in measurement play a critical role in the characterization of the measurement uncertainty. While the systematic approach proposed in [1, 2] is an appropriate tool to identify the transducing processes and to keep track of the points where uncertainties affect them, it has to be complemented with an encompassing framework enabling the identification of the points where idealization of real systems and realization of ideal systems operate: these may be important sources of measurement uncertainty. This analysis of the measurement problem and process can be exploited accordingly, to propose a preliminary, tentative reclassification of the sources of uncertainty, as follows.

Uncertainties due to idealization:

- on the object under measurement (e.g., modeling a physical object as a rigid body);
- on the measurand Q_{in}:
  - concerning the domain theoretical framework (e.g., modeling a material body as homogeneous, such that its density is constant; assuming a given set of influence quantities for Q_{in});
  - concerning the operational framework (e.g., neglecting the effects of some assumed influence quantities);
- on the measurement process:
  - concerning the domain theoretical framework (e.g.,, deciding the laws on which the measurement process is based);
  - concerning the operational framework (e.g., neglecting some of the assumed laws);
  - concerning the computational framework (e.g., admitting some approximations);
- on the transducer output quantity Q_{out}:
  - concerning the domain theoretical framework (e.g., assuming a given set of influence quantities for Q_{out});
  - concerning the operational framework (e.g., neglecting the effects of some assumed influence quantities).

Uncertainties due to realization:

- of the measurand Q_{in} (e.g., imperfect realization of its definition);
- of the measurement process:
  - concerning the environment (e.g., partial knowledge of the environmental conditions and their effects);
  - concerning the instrumentation (e.g., imperfect knowledge of the reference quantity values used in calibration; limited instrument stability);
- of the transducer output quantity Q_{out} (e.g., errors due to finite reading resolution).

On the whole, this classification might be exploited as a template in the construction of the uncertainty budget, to keep into account all the relevant components of the (combined standard) measurement uncertainty.

6. Conclusions

The aim of this paper has been to introduce the idea that modeling activities are fundamental to understand and assess the structure of the measurement process and to show how measurement uncertainties are essentially involved in the process: models are the ties between measurement and uncertainty. Since 1) any measurement is based on a specific set of interactions between the object under measurement and a measuring instrument and 2) any such interaction is ruled by a specific causal condition, as typically described by a physical law, the understanding of a measurement process is crucially dependent on the understanding of the transduction on which it is based. While in principle a purely black box model of the transducer behavior might be assumed, such a model is instead generally construed by exploiting the available physical theories and specifying them to the given measurement context and in view of the given measurement goals. In this modeling activity idealizations are constantly at work, so that all models can only be considered as tools to acquire approximate information on the actual systems and quantities involved in the process. These approximations are a critical source of uncertainties in measurement, and highlight that the GUM hypothesis that the measurand "can be characterized by an essentially unique value" is grounded on a biased consideration of the components of the uncertainty budget.

The foregoing analysis of the structure of the modeling activity involved in a measurement process appears to be significant for supporting in the identification of the uncertainty sources and in the evaluation of the amount of uncertainty due to any specific source. Indeed, the knowledge of the role of models in measurements and of the possible ways in which idealizations underlying the construction of models and their practical realization can affect measurement results is essential to appreciate the significance and the correctness of the information obtained by measurement. The inherent presence in any measurement process of both an empirical and an informational stage highlights that measurement results are not simply determined, but assigned consistently with the given model(s).

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