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On (kinds of) quantities

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Abstract

The third edition of the *International Vocabulary of Metrology* (VIM) has introduced the concept of kind of quantity as the "aspect common to mutually comparable quantities". While the concept is a fundamental one, as it is relevantly used several definitions throughout VIM, its definition is critical for several reasons. Not only "The division of the concept of 'quantity' according to 'kind of quantity' is to some extent arbitrary", as noted in the Vocabulary, but also the distinction between the concepts of 'quantity' and 'kind of quantity' is to some extent arbitrary. This paper discusses this subject, and suggests a possible solution to some of the issues identified.

1. Introduction

Quantities are a constitutive component of scientific knowledge, and a fundamental topic for measurement: "there are a few fundamental concepts in most, if not all, approaches to describing measurement. Probably the most fundamental concept pertains to the kinds of things that can be measured, i.e., quantities" [1]. Accordingly, their role is properly emphasized in the recent third edition of the *International Vocabulary of Metrology – Basic and General Concepts and Associated Terms* [2] ("VIM3" henceforth). Being usually deemed as specific properties (as in definition 1.1 of VIM3 – denoted as {1.1} for short from now on – "quantity": "property of a phenomenon, body, or substance, where the property has a magnitude that can be expressed as a number and a reference"), quantities unavoidably call for an epistemological ground for their characterization. The note 1 of the definition 1.1 of VIM3 – {1.1 N.1} – states: "The generic concept 'quantity' can be divided into several levels of specific concepts, as shown in the following table. The left-hand side of the table shows specific concepts under ‘quantity’. These are generic concepts for the individual quantities in the right hand column.” The left hand column of the mentioned table includes, in particular, length: therefore length is assumed here *to be a quantity*. On the other hand, the example 1 of the definition 1.2 of VIM3 – {1.2 Ex.1} – states: “The quantities diameter, circumference, and wavelength are generally considered to be quantities of the same kind, namely of the kind of quantity called length.”. Therefore length is assumed here *to be a kind of quantity*.

These premises, together with the acknowledgments that:

- quantities are defined as properties {1.1};
- kinds of quantities are defined as aspects of properties {1.2};
- properties and aspects of properties (whatever the term means) seem to be ontologically distinct entities,

lead to a conceptual inconsistency.

Of course, a straightforward solution is to assume that the term “length” is used here in a polysemic way, i.e., with (at least) two distinct meanings, for a quantity and a kind of quantity respectively. On the other hand, this interpretation, set aside its unusualness, is never justified in VIM3. In particular in the definition of “quantity dimension” {1.7}, and the related notes and examples, the relation between kinds of quantities and dimensions of quantities is clearly presented – “(...) quantities having the same quantity dimension are not necessarily of the same kind” {1.7 N.4} – so as to prevent the conclusion that the “aspect” that two or more quantities must have in common to be of the same kind is their dimension. Hence, it must be concluded that VIM3 loosely admits at least a partial interchangeability between the concepts of quantity and kind of quantity: at least in some cases, a quantity can be also a kind of quantity and/or a kind of quantity can be also a quantity. Our claim is that a few critical issues are implied in this hypothesis, also given the fact that the concept of kind of quantity appears in several definitions throughout VIM – “measurement unit” {1.9}, "ordinal quantity" {1.26}, "reference measurement procedure" {2.7}, "metrological comparability of measurement results" {2.46}, "measuring system" {3.2}, "reference measurement standard" {5.6}, "reference quantity value" {5.18} – just to mention some of them. The interpretation of some basic metrological concepts in the framework of the object-oriented paradigm will help us to point out such critical issues, and possibly to suggest a solution to them.

2. An object-oriented interpretation for quantities

The object-oriented paradigm is based on the distinction *class vs. object*. For example (loosely taken from the Collections framework, a structured subset of classes in the Java programming language), Vector is a class, whereas any given vector is an object of that class, i.e., a concrete instance of the concept of Vector. Class names will be capitalized from now on, to distinguish them from instance names. Hence, for example, a vector is an instance of (the class) Vector. It is interesting to note how this notational convention, recommended in the Java programming language, is similar to the statistical practice of denoting random variables by capital letters and their values in lowercase.

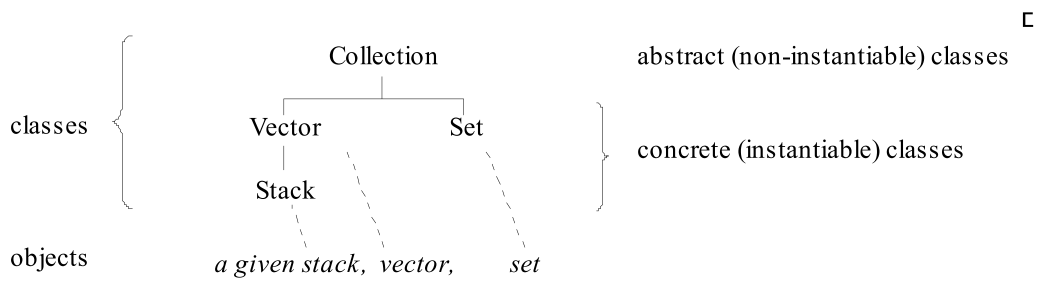
Objects are "terminal" entities of any structure which they belong to, in the sense that they cannot be in their turn instantiated. On the other hand, a structure can be defined among classes, which can be in an hierarchical relation with each other. For example, Stack is a subclass of Vector, in the sense that stacks can be implemented as specific vectors, and Vector is in its turn a subclass of Collection. Between a class and its superclass the relation IS_A holds: Stack IS_A Vector, and Vector IS_A Collection. Furthermore, this relation is transitive, a consequence of a feature called "inheritance" in object-orientation: if Stack IS_A Vector and Vector IS_A Collection, then automatically also Stack IS_A Collection. The fact that the nominally same relation holds between (i) objects and classes (a given vector IS_A Vector) and (ii) classes and superclasses (Stack IS_A Vector) is somehow misleading, but the distinction is fundamental and must be clearly maintained, as the extensional, i.e., set-theoretical, interpretation highlights:

object ∈ class (an object belongs to a class)

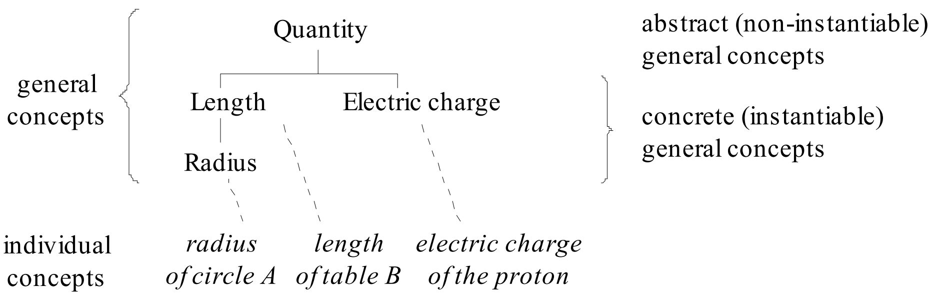
whereas:

class ⊆ superclass (a class is a subset of a superclass)

Important for our aims is also the distinction between concrete and abstract classes: a concrete class is a class that can be actually instantiated, i.e., a relation IS_A is directly allowed between an object and that class (in the previous example, a given object IS_A vector, hence Vector is modeled as a concrete class). On the other hand, an abstract class does not admit direct instances, as in the case of Collection, which cannot be instantiated except by generating an instance of one of its subclasses. Hence, a given object can in fact be the instance of a Collection, but only because it is an instance of this, e.g., Stack, or Vector, or Set. From this characterization, it follows that abstract classes must be superclasses in an object-oriented structure. Finally it should be noted that the distinction between concrete and abstract classes is by no means an "intrinsic" one – whatever this adjective means – and it largely depends on the aims and the state of knowledge of the subjects involved. For example, at a very preliminary stage of the modeling process the concept of Collection could be acknowledged as specific enough to be instanciable, i.e., corresponding to a concrete class, whose objects would then be characterized in a very generic way, and only as the result of a refinement analysis restructured as an abstract class. The following diagram synthesizes these concepts and their relations:



An analogous diagram can be drawn for quantities, by taking some exemplary items from the afore mentioned table in {1.1 N.1}, as follows:



This analogy could be further explored by considering objects as variables, i.e., containers for values. For example, a given *n*-dimensional vector stores as value an *n*-tuple of numbers, and an "individual quantity" such as the radius of a given circle "has a magnitude that can be expressed as a number and a reference", as the definition of "quantity" {1.1} states. Hence, a comparison of the two cases:

the object *v*	is a Vector	and has a value *x (= an n-tuple of numbers)*
the individual quantity *r*	is a Radius	and has a magnitude (expressed as) *y (= a number and a reference)*

clearly shows that "magnitude" (a non-defined concept in VIM3) could be simply thought of as a particular case of value for those particular cases of variables that are individual quantities. Were it so, why not simply remove the term "magnitude", and replace it by the more customary, and general, "value"? In this view it is interesting to take into account the French definition of "quantity": "propriété d'un phénomène, d'un corps ou d'une substance, que l'on peut exprimer quantitativement sous forme d'un nombre et d'une référence". A comparison of the English and the French definitions shows their asymmetry:

- the English definition implies a ternary relation: a quantity is (1) a property having (2) a magnitude expressed as (3) a number and a reference;
- the French definition implies a binary relation: a *grandeur* is (1) a *propriété* expressed (quantitatively) as (2) *un nombre et une référence*.

Since the adverb *quantitativement* in the French definition does not convey any actual information, the French definition itself highlights the basic structure of the definition: a quantity is something expressed by something. It should also be noted that VIM3 implicitly uses the concept of property value, while not defining it, as in: “A nominal property has a value, which can be expressed in words, by alphanumerical codes, or by other means.” {1.30 N.1}. Hence, the removal of the controversial term “magnitude” would lead to the further benefit of requiring a restatement for the concept of nominal property, currently troubled by the fact that such “other means” in principle include “numbers and references”, thus implying that quantities are specific cases of nominal properties, quite a peculiar standpoint.

A further result of the analogy synthesized in the previous table is to highlight the structural, if not ontological, distinction between classes and objects, and therefore between “general” quantities and “individual” quantities. In this regard the second edition of VIM [3] (“VIM2” henceforth) acknowledged that “the term quantity may refer to a quantity in a general sense or to a particular quantity” {VIM2 1.1, N.1}, and from this assumption tried to carefully distinguish between the two (sub)concepts. On the contrary, VIM3, while admitting that “the generic concept ‘quantity’ can be divided into several levels of specific concepts” {1.1 N.1}, seems to forget this “division” in its terminology (and the choice of the term “division” as related to concepts is rather questionable: while a set is divided into its subsets, a concept is specified by its subconcepts), and simply leaves to the reader the task of discriminating whether a particular occurrence of the term “quantity” refers to “the general sense” or the “individual” concept. Consider, for example, the definition of “system of quantities” given by VIM2 – “set of quantities, in the general sense, among which defined relationships exist” – and VIM3 – “set of quantities together with a set of non-contradictory equations relating those quantities”. Only the former explicitly states that such relationships / equations hold among “general” quantities. An even more cogent example is given by the definition of “measurand”, which in VIM2 is “particular quantity subject to measurement” – and not “quantity subject to measurement” as incorrectly stated in {2.3 N.2} – and in VIM3 is “quantity intended to be measured”. Given the acknowledged requirement that a measurand must be defined – consider, for example, the definition of “true quantity value” {2.11}: “quantity value consistent with the definition of a quantity" – should such a definition refer to a "general" quantity or an "individual" one?

3. Quantities and kinds of quantities

Why was this ambiguity introduced? An answer can perhaps be found in the fact that the term "kind of quantity", only mentioned in {VIM2 1.1 N.2}: "Quantities that can be placed in order of magnitude relative to one another are called 'quantities of the same kind'"), is defined in VIM3 as "aspect common to mutually comparable quantities" {1.2}. A hypothesis could be made that such "kinds of quantities" somehow play the role that in VIM2 was covered by quantities "in the general sense". The given examples (the above-mentioned example 1, and the similar example 2, "The quantities heat, kinetic energy, and potential energy are generally considered to be quantities of the same kind, namely of the kind of quantity called energy") explicitly support this position, and so does the fact that the term "kind of quantity" is used, for example, in the notes associated with the definitions related to dimensional analysis ({1.7} "quantity dimension"; {1.8} "quantity of dimension one"). On the other hand, the reference to "mutually comparable quantities" in the definition is quite ambiguous (and it should be noticed that the concept of comparison is not defined in VIM3, despite its importance). In particular, {2.1 N.2}, "measurement", states that "measurement implies comparison of quantities", and in this case "individual" quantities are plausibly concerned. Is this concept of comparability between "individual" quantities inherited by the comparability between quantities "in the general sense", i.e., if classes are comparable then are their instances so? Or, vice versa, is VIM3 ambiguous for what concerns comparability, and therefore kinds of quantities?

To explore the issue of a possible ambiguity in the concepts of kind of quantity and comparison of quantities, let us provisionally write:

kind_1	to denote	a kind of quantities "in the general sense" e.g., Length as superclass of Diameter and Circumference

kind$_2$		a kind of "individual" quantities e.g., Radius as class of which radius of circle A and radius of circle B are instances, but possibly also Length as class of which diameter of circle A and circumference of circle B are instances

and:

comparison$_1$	to denote	a comparison between quantities "in the general sense" e.g., Diameter and Circumference, which are indeed dealt with in the same way from a dimensional point of view
comparison$_2$		a comparison between "individual" quantities e.g., radius of circle A and radius of circle B, but possibly also diameter of circle A and circumference of circle B because of their common dimension

If the definitions of "kind of quantity" {1.2}, together with its example 1, and "measurement unit" {1.9} are compared accordingly, we have:

A	{1.2} kind of quantity	aspect common to mutually comparable quantities
B	{1.2 Ex.1}	the quantities diameter, circumference, and wavelength are generally considered to be quantities of the same kind, namely of the kind of quantity called length
C	{1.9} measurement unit	real scalar quantity, defined and adopted by convention, with which any other quantity of the same kind can be compared to express the ratio of the two quantities as a number

It seems then reasonable to assume the following hypotheses.

- While A could refer, in principle, to both kind$_1$ and kind$_2$, and to both comparison$_1$ and comparison$_2$, it is indeed aimed at defining kind$_1$ by referring to comparison$_1$. As a counterexample, consider that radius of circle A and radius of circle B are surely comparable (individual) quantities: if A also dealt with this case, Radius would become a kind$_1$, and in consequence "kind of quantity" and "quantity" would be synonymous terms.
- Accordingly, B refers to kind$_1$ (and thus implicitly to comparison$_1$), as already discussed.
- On the other hand, C refers to kind$_2$ and comparison$_2$.

Hence, only comparisons between quantities "in the general sense" must be taken into account to define the concept of kind of quantities. If this is the case, a clarification in this sense would be a significant help in removing such an ambiguity.

Finally, one more critical issue can be pointed out relating to the very definition of "kind of quantity" as an "aspect" (again a non-defined term in VIM3). Under the hypothesis that the model underlying the concepts of quantity and kind of quantity in VIM3 is not too different from the ("object-oriented-like") one sketched here, kinds of quantities seem to play the role of (plausibly abstract) superclasses in an object-oriented hierarchy. Accordingly, a superclass has, or gathers, or maintains, the commonality of its subclasses, but it *is not* such a commonality. That "aspects of *x*" and *x* are not homogeneous when *x* is a physical object is quite obvious. For example automobiles and bicycles share the "aspect" of being able to transport people: this "aspect" could be used to characterize, and even define, their superclass Vehicle, but a vehicle has the ability to transport people, but is not that ability, and therefore is not that aspect. Vice versa, the above-discussed hypothesis of the partial interchangeability between the concepts of quantity and kind of quantity implies that "aspects of quantities" (whatever this means) are homogeneous to quantities. Even acknowledging the highly controversial ontology of properties ("what is a property" is a very complex issue), and thus of quantities, the definition of "kind of quantity" given by VIM3 appears hard to understand. Is Length really what, for example, Diameter and Circumference have in common? The object-oriented structure sketched in this paper shows perhaps a more convincing standpoint: Length is a quantity (in the general sense) that subsumes the quantities (in the general sense) Diameter and Circumference as particular cases. From an extensional point of view, a kind of quantity is not an aspect common to a given set of quantities, but is that set, that can be internally structured in a hierarchical way and whose "top" element in that case can be chosen as representative for the whole set, i.e., the kind itself – in the example above, the set named "Length".

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