preprint of a paper published in
Measurement, 21, 3, 79-90, 1997, doi:10.1016/S0263-2241(97)00050-X

The role of determination and assignment in measurement

Luca Mari
Libero Istituto Universitario “C.Cattaneo”,
C.so Matteotti, 22 - Castellanza (VA), 21053 ITALIA
Tel: +39 331 5721 - Fax: +39 331 480746 - Email: lmari@liuc.it

Abstract

What characterizes measurement with respect to generic evaluation? The representational model to measurement theory seems to underestimate this problem, of primary importance to justify the claimed objectivity of measurement. Indeed measurement has the twofold nature of an empirical and a symbolic operation, so that both these components must be taken into account in its formalization, which should emphasize the central role played by measuring systems in qualifying the operative character of measurement. From this perspective, the paper suggests that the peculiarities of measurement as a specific form of evaluation can be meaningfully interpreted in terms of the distinction between determination and assignment. After an analysis of the basic characteristics of these concepts and their relations, the paper describes a functional model of a measuring system and proposes a formalization for the concept of measurement that is based on such a model and specifies the representational point of view. Hence, measurement assumes the connotation of a homomorphic evaluation realized by means of a measuring system.

Keywords: *Measurement theory; Modeling of measuring systems; Measurement as determination and assignment*

1. Introduction

What gives measurement its recognized role of fundamental tool to acquire an objective knowledge on things? Such a question is so general and basically related to the nature of measurement itself that we can be sure that a defined answer (if not a whole set of them) was given in the past, thus causing us to wonder why such an answer cannot be assumed as valid also today.

A traditional position on the epistemic status of measurement does exist, and is fundamentally based on the assumption that «each thing that can be accessed through our knowledge possesses a number, since without numbers we can neither understand nor know» (translated from an excerpt of Pythagorean school, about V century b.C.). In the same sense, «the elements of numbers were supposed to be the elements of all things, and the whole heaven a musical scale and a number» (translated from Aristotle, *Metaphysics*, about 350 b.C.). The formulation given to this hypothesis by Galileo Galilei is known: according to him the “great book of nature” cannot be understood «but by learning its language and knowing the characters in which it is written: it is written in mathematical terms» (translated from G.Galilei, *Il Saggiatore*, 1632).

This view had a substantial part in the foundation of the theory of errors in measurement, formulated by C.F.Gauss at the beginning of the XIX century. Such a theory was based on the concept of a *true value* that, even though not known, and maybe in principle recognized as unknowable, characterizes an attribute (for the sake of generality the term “attribute” will be used here instead of “quantity”, “observable”, “parameter”, …; cf. [1], where the concept of attribute and the problems related to the definition of attributes are analyzed in a metrological perspective). Measurement would be then aimed at finding an estimate for such a value. The meaning of measurement was thus founded on the hypothesis that numbers *are in* the world, and can therefore be “extracted” by a suitable empirical operation.

Such a view has been more and more widely criticized in this century. R.Carnap [2], for example, asserts that «a phenomenon does not contain anything of numerical, but only our sensation. We can introduce numerical concepts by establishing a procedure to measure them. The numbers are assigned to the nature by ourselves, because phenomena exhibit only the qualities we observe». The ontological view of a true value to be determined by measurement was going to be replaced by a more pragmatic interpretation, according to which «measurement is the assignment of numerals to objects or events according to rule, any rule», as S.Stevens [3] wrote.

This shift of perspective solved the basic criticism against the classical point of view to the concept of measurement, i.e., the fact that it requires the use of the concept of true value that cannot be operationally maintained, while generating a new kind of problems on the nature of measurement. In the new interpretation of the concept, nothing justifies the usually asserted hypothesis that measurement is an *empirical* operation producing *objective* results. The distinction between measurement and a generic symbolic evaluation would simply disappear. If the accordance to "any rule" is the requirement that characterizes measurement, then its results may inform us on the measurer's view on the thing under consideration, but, generally speaking, surely not on the state of the thing itself. What would then justify the scientific, technical, and social significance of measurement? Provided that to measure is a way to evaluate, what would distinguish measurement from generic evaluation?

In this work the epistemic role of measurement will be analyzed, in the perspective of the alternative between the classical, ontological, position and the current, pragmatic, one. Given the basic hypothesis that such positions can be interpreted in terms of the alternative between the concepts of determination and assignment, an intermediate point of view will be assumed, asserting that measurement should be regarded as partly a determination *and* partly an assignment. This will allow the analysis of the constitutive role of measuring systems and a revised definition of the concept of measurement itself.

2. Measurement as determination or assignment

For many years measurement was thought of as an operation appropriate to evaluate exclusively extensive, additive, physical attributes (and the terminology "weights and measures" is still a relic from the past, when measurement had an even more specific meaning, specifically related only to geometrical attributes). In this context the interpretation of its absolute and objective meaning arose.

More recently, a specific attention has been devoted to such an operation by social and psychophysics scientists, who were mainly interested in establishing procedures to assess some, often subjective, characteristics in an unambiguous and thus intersubjectively comparable way. However, while widening the field of applicability of measurement, such studies in their extreme consequences eventually assumed a completely conventional interpretation, according to which any evaluation verifying some formal rules can be aptly considered a measurement. For example, for a measurement in a nominal, classificatory scale the only rule is that a single value be assigned to each thing: that is why S.Stevens could assert the sufficiency of "*any* rule".

Measurement was losing its foundations on the metaphysical and empirically unknowable concept of true value. But, on the other hand, such foundations had to be found somewhere else, in order to avoid the somewhat embarrassing conclusion that "measurement" is simply a different name for "evaluation", nothing being left to justify the requirement of its objectivity. Between such two opposite views, a whole range of definitions of measurement can be found in the scientific and technical literature. In our view, many of such definitions can be categorized in two wide semantic classes, partly overlapping, whose representative elements are, for example, the following:

* «*measurement is the set of operations having the object of determining the value of a quantity*» [4]

for the first class, and:

* «*measurement is the process of empirical, objective assignment of numbers to the attributes of objects and events of the real world, in such a way as to describe them*» [5]

for the second one.

We suggest that the distinction between such two definitions be expressed in terms of the basic question: *is measurement a determination or an assignment*?

Such a question should not be regarded as simply nominal. The result of a measurement is thought of as *determined*, i.e., "extracted" and expressed in formal terms, whenever the obtained value is considered an intrinsic property of the thing, ontologically existing in the thing independently of any interaction of the thing with a measurer. It can be aptly called the true value of the thing for the attribute under measurement. The basic criterion to evaluate the quality of a measurement-determination (hereinafter it will be denoted as "measurement_d" for short, to distinguish it from measurement-assignment, "measurement_a") is then the degree of approximation of its result to the true value. A difference between such a true value and the result of a measurement_d is considered and formally dealt with as an "error" in the determination. According to this interpretation, then, «on the basis of a reflection on the meaning of the obtained results of the measurement, the experimenter thinks about the true value, the value that the best possible instrument would have generated» (translated from [6]).

From an historical point of view, at least two different reasons contributed to a criticism of this position.

One came from the critique of the operationalism, which judges as scientifically acceptable only those concepts which can be defined in terms of sets of operations, and whose meaning is given by such operations [7]. In this view, one is led to recognize that the concept of true value of an attribute is not operational, but in the questionable situation in which an operative definition can be established of a "true measurement", i.e., a procedure able to produce a value that is true by definition (a tentative proposal in this sense is in [8]). Among the difficulties arising from such an assumption there is the problem to justify the experimental variability of measurement results.

A second reason leading to a criticism of the traditional objectivistic position arose from the attempts to widen the field of application of measurement to non-physical attributes, thus highlighting its analogies with other forms of evaluation, such as estimation and judgment of preference. This position emphasized a *subjective* component of measurement, recognized in the unavoidable presence of a measurer's judgment in reaching a measurement result. Measurement becomes an activity of *decision making* [9] and assumes the epistemic status of an *assignment*, thus eliminating the traditional condition of the necessary pre-existence of a true value.

Such a distinction between the concepts of measurement_d and measurement_a needs further consideration and, for the sake of clarity, in the following paragraphs it will be discussed in its most extreme interpretations, although a wide range of intermediate positions can be assumed.

2.1. Determination and assignment: an example

The distinction between determination and assignment is empirical rather than formal: both determination and assignment can be formally expressed as evaluations, i.e., operations aimed to associate a value with the thing under consideration.

An example can help us make this point clear (see also [3] where a similar example is discussed). A coach wants to associate a number with each player of his soccer team. Formally this can be expressed as an operation *op* such that *op*(*player*)=*number*. Is *op* a determination or an assignment? The answer cannot be given but considering the aim of *op*, and how the operation is empirically performed. The coach may want to plan a strategy for his team. This will lead him to establish that a given player will have a given role, and then a given number (assuming that each role is identified by a different number). In this case such an operation is what the Philosophy of Language would call an "initial baptism", i.e., the choice of a lexical term acting as the identifier for a thing that did not have such a term previously associated with. This is an assignment. However, the coach may also want to verify during a game if a certain player for which a given number was previously established wears the shirt with the correct number. To do so, the coach has to observe the player and read the number on his shirt. This is a determination.

In both cases the operation can be formally expressed as shown, but the differences in the empirical realization and information content are apparent.

2.2. Descriptive and normative assertions

As for any symbolic evaluation, the formal definition of a measurement requires the explicit statement of the conditions that must be verified by the evaluation. The weakest condition that can be assumed is simply that whenever two things are regarded as equivalent with respect to the considered attribute then the same symbol must be associated with them. On the other hand, stricter conditions can be imposed. For example, whenever the thing x_1 is regarded as "empirically greater" than the thing x_2 with respect to the considered attribute then the symbol associated with x_1 must be in relation "greater than" with the one associated with x_2. According to the representational model (cf., e.g., [10]), such conditions are expressed as axioms and univocally characterize a given theory of measurement, and therefore a scale of measurement. However, as already pointed out in [11], the problem remains whether these axioms should be interpreted as conditions on either the described things or the way such things are described, i.e., whether they have a *descriptive* or a *normative* meaning.

A well-known example of this alternative comes from the interpretation of the probability theory axioms, e.g., $P(x_1 \cup x_2)=P(x_1)+P(x_2)-P(x_1 \cap x_2)$ for any two subsets/events x_1, x_2. Does such an axiom describe a characteristic of x_1 and x_2, or does it express a rationality condition in their evaluation? The first answer implies a descriptive meaning for the axiom, asserting "how the world behaves", whereas the second one suggests a normative meaning, specifying "how the subject should decide" in presence of uncertainty.

The distinction between measurement$_d$ and measurement$_a$ can be interpreted in analogous terms. A theory of measurement$_d$ is *descriptive*, since the correctness of measurement results is thought to depend on their closeness to the true value. A theory of measurement$_a$ is based on the *normative* assumption that this correctness has to be evaluated in terms of the satisfaction of given formal rules. This implies that the sentence expressing the result of a measurement$_d$ is assumed to have a given, although possibly unknown, truth value associated with it. On the other hand, the result of a measurement$_a$ is not subject to truth evaluation but can be

recognized as more or less adequate to a given goal. Therefore: is a sentence expressing a measurement result more or less *true*, or more or less *adequate* to a goal?
Interestingly, the same issue can be considered in terms of the function recognized to the linguistic act related to the expression of measurement results: referring to the distinction suggested in [12], has such an expression a *descriptive* function, thus being more or less true, or a *signaling* one, thus being more or less efficient?

2.3. Relations with the demarcation problem

The distinction between the concepts of measurement$_d$ and measurement$_a$ can be considered in parallel to the problem of demarcation between *empirical* (or experimental) sciences and *formal* (or theoretical) ones (cf., e.g., [13]). Accordingly, a science can be considered formal whenever its assertions can be validated on the basis of axiomatic-deductive methods uniquely, whereas in the case of experimental sciences some empirical operation is also required. While measurement$_d$ inherently belongs to the empirical world, because of its claimed objectivity, measurement$_a$ is characterized in formal, and even abstract (as in [14]), terms. In this sense, in the context of the representational model the usual assertions requiring that measurement be a «process of empirical, objective assignment» (as in the quoted definition of [5]) seem to be extrinsic and not to influence the formalization in significant way.

The analyzed distinction between the concepts of determination and assignment, and correspondingly the historical shift from measurement$_d$ to measurement$_a$, is so radical that one could even think of it in terms of a *scientific revolution*, in the sense proposed by [15]. Indeed, this shift does have some peculiar characters of a revolution, e.g., the fact that the same terms, like "measurement", "quantity", "measuring system", …, are used by different groups of researchers with different meanings. That is why we believe that it can be considered as an actual paradigm substitution. Are such two paradigms incommensurable? Is measurement a determination or an assignment?

3. Interlude: on the epistemic role of the measurement

In [16] T.Kuhn expresses some «skepticism about the two predominant descriptions on the function of measurement», asserted to be the confirmation of already formulated theories and the exploration of new theories (he is apparently taking into account only the *scientific* function of measurement). Explicitly: does a measurement produce a result that can falsify a theory?
Such a question, deeply rooted in any epistemology, refers to the presumed primitiveness of measurement results as data of knowledge: if a measurement result can be obtained by purely empirical means and independently of any influence of the measurer, then it could be assumed as an "objective" reference to confirm or to confute an hypothesis. The Kuhn's skepticism is plausibly addressed to this traditional view of the epistemic role of measurement, according to which measurement constitutes a "protocol of truth". In the neo-positivistic interpretation a "scientific view of the world" can only be based on knowledge «that can be reduced to elementary assertions about sensible data» [17]. But can measurement results be considered as such "sensible data", basic elements in the construction of scientific reasoning?
According to [18], «any measurement, even the most straightforward one, must be based on some theoretical assumptions, on "principles", "hypotheses", or "axioms" that measurement itself cannot deduce from the sensible world, but that has to refer to such a world as postulates of thought». Measurements (and not only

generic evaluations) would be thus "charged with theory", since «any observation necessarily includes a phase of data elaboration. Strictly speaking, purely observational terms simply do not exist » (translated from [19]).

This issue about the primitiveness of the measurement results appears here as a re-expression of the already highlighted alternative: measurement as determination (operation of only empirical nature) or assignment (operation "charged with theory")?

The current prevalence of the measurement$_a$ paradigm can be at least partly put down to the contemporary climate of epistemic relativism, characterized by the substitution of the objectivistic concept of truth with the subjectivistic criterion of the consensus. In this context a new conception of theory arose: the previous connotation, a set of assertions having a truth value and bringing an objective knowledge, has been replaced and now theories are thought to be tools whose cognitive status inherently depends on historical and social conditions and is relative to the system of beliefs of the particular scientific community who produces them. As a consequence, alternative paradigms are supposed to be incommensurable with respect to objective criteria [15].

Even in such a frame, measurement can be hardly considered as producing results pertaining only to the knowledge status of the measurer, as emerges for example when we try to justify the efforts of the search for precision in measurement (this argument is analyzed in detail in [20]). Indeed, «delicate measurements have always contributed to the progress of our knowledge in physics not to a speculative *philosophia naturalis*, but a quantitative description of nature» [21]. Measurements done by Cavendish, Coulomb, Fizeau, Michelson, Millikan, only to mention a few of the classic experiments performed in the history of physics and based on "delicate experiments", confirm that some characteristics of nature come to light only by means of precise observations. What in such measurements had been obtained was found only because of their precision (in this context [22] is maybe worth quoting as a counterbalance: «do not ever point to a precision greater than the one the problem under examination requires. That is why I have no confidence in precision: I believe that simplicity and clarity are values in themselves, but that precision and exactness are not»). Measurer's subjectivity plays an unavoidable role in the evaluation of the quality of measurement results, and thus in the choice of which results should be considered as meaningful. But an actual measurement (contrasted to a generic evaluation) seems to produce results that in a way impose themselves to the measurer.

In the search for a non purely subjectivistic interpretation of measurement, two main reasons of conventionality must be however recognized as present. First, measurement results are symbols, linguistic entities, and as such they inherit the conventional nature of language. Secondly, measurement does not lead to absolute evaluations, since its results are referred to a given standard, e.g., the unit of measurement in the case of ratio scale attributes (interestingly, neither of the quoted definitions highlight this point).

We suggest this second reason of conventionality, the necessary presence of a reference, to be the conceptual starting point to build the "objective component" of measurement. Once it is agreed that to measure is to evaluate with respect to a standard, then the claim that measurement is an "objective" and "empirical" process (cf. the quoted definition by [5]) can be made explicit as:

(C1) *the standard adopted in the measurement operation must be well-defined and external to any specific measurer*;

(C2) *the operation of* (direct or indirect) *comparison of the thing under measurement to the standard must be well-defined and carried out independently of any specific measurer*.

In other words, we are proposing to consider (C1) and (C2) as *required* conditions for an evaluation to be called a measurement.

On the other hand, the character of ideality of such conditions is apparent: more than a given status, the characteristics of being objective and empirical appear to be a target point, to be reached via successive approximations. This implies the enhancement in the definition of the standard and the measurement procedure, and the reduction of the measurer influence on the thing under measurement. In this sense the entire scientific and technological development of metrology can be understood as the effort to reach *more and more objective and empirical* evaluations.

4. Measurement as determination *and* assignment

Measurement is currently conceptualized and formalized in terms of the so-called *representational model*. Any measurable attribute is thought of as characterized by a defined set of observable qualitative relations that its evaluation induces on the set of things under measurement. For example, the evaluation of temperature would be based on the prior recognition that things for which such an attribute is measurable can be ordered with respect to their temperature state. Whenever a thing x_1 is (qualitatively) felt warmer than a thing x_2 the value *temperature*(x_1) associated with x_1 must be (quantitatively) greater than the value *temperature*(x_2) associated with x_2 (for a critical analysis of the usual interpretation of the concept of attribute within the representational approach see [1]).

(A note on the term "thing" here adopted is required. In our view, *things*, i.e., objects, processes, events, …, are characterized at any time by the fact that they are in a given *state*. While holding their own individuality over time, they can however change something of their appearance and behavior. Things should be then formalized as mappings of a time set to a state set, and arguments of attributes should be states, i.e., "time instances" of things, instead of things. For the present discussion the distinction between things and their states is inessential, and therefore will no longer be considered).

A measurement is then formalized as a *morphism*, i.e., a mapping $\underline{f}:\underline{X}\to\underline{Y}$, $\underline{f}=<f, f_R>$, of a set $\underline{X}=<X=$set of things under measurement, $R_X=$set of relations defined on it$>$ to a set $\underline{Y}=<Y=$set of values for the attribute under measurement, $R_Y=$set of relations defined on it$>$ with the condition that any relation in R_X among elements in X *must be conserved* by $\underline{f}$, so that if x_1, x_2, $\ldots\in X$ are in the relation $r\in R_X$ then correspondingly $f(x_1)$, $f(x_2)$, $\ldots \in Y$ must be in the relation $f_R(r)\in R_Y$. A mapping for which such a property holds is called a *homomorphism*, and entities such as $\underline{X}$ and $\underline{Y}$ are called *relational systems*.

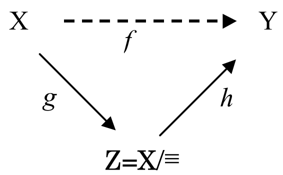
The formal correctness of the representational model is not under discussion (although the single-valuedness of the mapping f makes the assumption of perfect exactness of measurement explicit, and therefore highlights a limitation of the formalization. The extension to the general case of inexact measurement is still largely an open issue and will not be considered here), nor can its conceptual merits be underestimated. In our view, the greatest one of them is to highlight the nature of measurement as an operation aimed to generate symbols that can be considered as faithful substitutes of the corresponding measured things [23]. Indeed, instead of empirically operating with things one can formally deal with their symbols, assured that conclusions drawn on symbols, e.g., *temperature*(x_1)>*temperature*(x_2), are also valid for things, e.g., x_1 is warmer than x_2.

We believe that an "abstract" view of measurement, such as the one basing the approach of [14], cannot however fully comprehend the *operative nature* of the concept: as [24] has been already pointed out, any

interpretation of measurement only stressing its formal aspects is bound to fail in distinguishing measurement from generic (homomorphic) labeling, and, even more generally, any (unambiguous) naming operation.
The purely formal interpretation of measurement arisen within the currently dominant paradigm of measurement_a does not seem to justify the asserted role of measurement as a tool of objective knowledge. This epistemic character of measurement cannot be solely derived from the formal requirement that measurement be a homomorphism, to be considered rather as a self-consistency condition.
Even an abstract theory of measurement cannot abstract from the conditions (C1) and (C2) stated in the previous Section, but at the price of becoming a theory of (homomorphic) evaluation. The operative nature of measurement has a relevant part also in the foundation of the concept and its formalization.

4.1. Measurement and measuring systems

Conditions (C1) and (C2) can be interpreted in terms of the foundational hypothesis that measurement is a form of evaluation performed by means of a *measuring system* (MS). As a matter of fact, in different fields different concepts of MS are usually adopted so that, e.g., mechanical, electrical, and systemic paradigms of MS have been proposed: MSs as physical instruments whose moving parts have a relative position that can be controlled; MSs as black boxes converting signals from input to output and characterized by a transfer function; MSs as systems with a reference state and whose state transitions depend on the interaction with the thing to be measured. A general characterization of the concept, such that both a dynamometer and an IQ test to be considered (part of) MSs, should highlight the *functional meaning* of measurement and thus the *functional role* of MSs.
Any MS *realizes the mapping* f, i.e., implements an attribute associating a symbol $y=f(x)\in Y$ with a thing $x\in X$. The application of any function $f:X\rightarrow Y$ induces on its domain an equivalence relation "≡" such that $x_i \equiv x_j$ iff $f(x_i)=f(x_j)$. Then f can be thought of as resulting from the composition $g \circ h$ of a function $g:X\rightarrow Z$, mapping X to the "reduced set" Z=X/≡, i.e., the set of the ≡-equivalence classes, and a function $h:Z\rightarrow Y$, mapping X/≡ to Y, $f(x)=h(g(x))$.



With respect to f, x is informationally equivalent to (i.e., provides exactly the same information as) any other x' belonging to the same equivalence class $z=g(x)$, $y=h(z)=h(g(x))$ being the symbol associated with such an equivalence class. The information quantity conveyed by y is minimum if #Z=1, i.e., the discrimination capacity of f is null, and is maximum (in the case of uniformly distributed a priori probability) whenever g is injective, i.e., the discrimination capacity of f is maximum relatively to the given set X (cf. also [25]).
The task of any MS is to associate a symbolic entity, assumed as measurement result, with the thing under measurement, thus generating a link between the empirical realm of things and the informational realm of symbols, i.e., a link between what K.Popper calls the *world1* («the world of the physical entities») and the *world3* («the world of the products of the human mind») [12]. This twofold nature of MSs (and fundamentally of measurement) is reflected in their basic functional structure, which always includes an empirical, "thing-oriented", component and an informational, "symbol-oriented", one. These subsystems will be called for short *acquisition component*, AC, and *presentation component*, PC, respectively.

The AC is aimed at interacting with the thing under measurement, with the threefold function of (*i*) *filter*, (*ii*) *comparator*, and (*iii*) *classifier*.

(*i*) Given the (a priori infinite number of) attributes by which the thing can be characterized, the AC generates an output depending on a single attribute, the measurand, or a small number of attributes, the measurand and some "influence quantities" (for the sake of simplicity the so-called "multi-sensing devices" will not be taken into account in the present analysis). Different measurands require different ACs, and the nature of the AC depends on the nature of the measurand. For example, IQ measurement requires an AC able to interact with persons and to produce an output that abstracts from all attributes that can be evaluated on persons but their IQ, as typically is claimed to perform a multiple choice test, the AC output being here the set of correct answers given by the person.

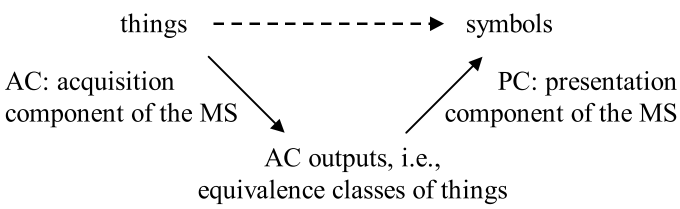
(*ii*) The AC realizes a (direct or indirect) comparison of the thing under measurement with a given standard reference, so that its output empirically expresses the relation between the thing and the reference. In the example, the AC is the test schema, i.e., the specification of the number of questions included in the test and their structure, whereas the reference is the test content, i.e., the actual set of questions. As a general fact, given the same thing and AC, the AC output changes whenever a different reference is chosen.

(*iii*) In their interaction with the AC different things can generate the same output: the application of the AC induces then an equivalence relation on the set of things under measurement. Two things are classified as AC-equivalent whenever they generate the same AC output, each AC-equivalence class of things being associated with a given AC output. In the example, two people who have correctly answered the same set of test questions are considered IQ-equivalent, and one of such sets is associated with each IQ-equivalence class of persons.

In its behavior an AC thus realizes a *n*-to-one mapping, formalized by a function *g*.

The PC is then aimed at associating a symbol with the AC output, i.e., any given AC-equivalence class, thus realizing a one-to-one mapping in which the information on the empirical relation between the measured thing and the reference is expressed formally. Such a mapping is formalized by a function *h*.

The whole measurement operation results from the composition of such two mappings:



that can be trivially generalized to morphisms in the case domain and codomain of the mapping things→symbols are in fact relational systems.

Before analyzing in some more detail this model – it will be called an *operational-representational model* for reasons that will be made clear later – and its consequences for the concept of measurement, let us make our position explicit:

* we suggest that the representational model can be understood as a formalization of the mapping $\underline{f}$:(relational system of things)→(relational system of symbols) abstracting its empirical structure, expressed by the composition $\underline{g}\circ\underline{h}$, and therefore that the present model *specifies* the representational one to the peculiar case of measurement;

* we suggest that the AC output can be considered as *determined* by the AC and that PC output can be considered as *assigned* by the PC, and therefore that measurement is both a determination and an assignment.

4.2. A critique to the representational model of measurement

The representational model asserts that an evaluation can be aptly called a measurement whenever it conserves the relations defined on the set of evaluated things: «the modern form of measurement theory is representational: numbers assigned to objects/events must represent the relations perceived between the properties of those objects/events» [26]. This means that *before* measurement such relations should be known in their extensive definition, i.e., as the explicit list of those n-tuple of things belonging to each n-ary relation. The knowledge on things under measurement would be elicited in finding the qualitative relations among things, and thus before measurement. The only role of measurement would be of (homomorphic) symbolic expression of such an already obtained knowledge.

This model is relevant to situations of evaluation by human subjects, without the mediation of any external MS. In such cases it imposes a formal condition that guarantees the meaningfulness of the values assigned as evaluation results (a strong support to the development of the representational model came in fact from disciplines such as Psychophysics and Econometrics, for which MSs as physical devices are usually unavailable). On the other hand, whenever the mappings $\underset{\sim}{g}$ and $\underline{h}$ are realized by the AC and PC of a MS respectively the condition loses a big part of its practical importance. In the design of a MS some knowledge is required on the *form* of the qualitative relations assumed on the set of things, so that the correctness of the MS behavior can be verified, but surely not on the *specific elements* of such relations, i.e., their extensive definition.

In reference to IQ evaluation, while the transitivity of the relation "has a higher IQ degree than" is hypothesized as part of the very concept of IQ and thus implemented in the structure of the IQ test, nothing is in general known before measurement on the truth value of the propositions "x_i has a higher IQ degree than x_j", for any given x_i and x_j. In some cases, the IQ example is plausibly one of them, the only way to assess such truth values, i.e., to evaluate whether x_i and x_j are in the given relation or not, is to measure them, and then to compare the corresponding measurement results. Therefore there are no "perceived relations" among things before measurement is performed. A radically operationist position could even be maintained ("IQ is the attribute evaluated by the IQ test"), and the evaluation results can be validated only *a posteriori*, if the measurand appears connected in a formal relation to other attributes that can be independently evaluated.

4.3. An operational-representational model of measurement

The knowledge available on the measurand before measurement is primarily implemented in the AC, chosen so that on its output set Z a set R_Z of relations is empirically defined in correspondence with the relations assumed in R_X, i.e., $r \in R_X$ and $g_R(r) \in R_Z$ have the same form. For example, if the knowledge on the concept of IQ leads to assume r="has a higher IQ degree than" to be a transitive relation then the AC must implement a transitive relation $g_R(r)$ (e.g., "has correctly answered a number of test question greater than").

The basic difference between relations in R_Z and in R_X is that before measurement the former are completely known, being part of the empirical definition of the MS, while the latter are in principle *not* extensively known, as previously considered. It is the factual realization of measurement that leads to identify which elements are in relations in R_X: things are found to be in a relation r whenever they generate AC outputs in relation $g_R(r)$. Hence the mapping $\underset{\sim}{g}$ is a homomorphism by construction, and its property of conserving the relations defined on its domain is assumed as a consequence of the empirical nature of the AC instead of requiring to be verified.

To complete the measurement, each AC output has to be labeled by a symbol, taken as measurement result. The relational system $\underline{Y}$ is chosen so that the information obtained in the acquisition phase is *integrally* expressed in symbolic terms. This is formalized by requiring the mapping $\underline{h}:\underline{Z}\rightarrow\underline{Y}$ be a monomorphism, i.e., an injective morphism. The injectivity of $\underline{h}$ makes explicit that everything that can be discriminated from the empirical point of view, $g(x_i)\neq g(x_j)$, should be kept as distinguishable also in symbolic terms, $h(g(x_i))\neq h(g(x_j))$.

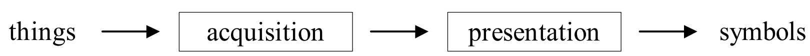
If $\underline{g}$ is a homomorphism and $\underline{h}$ is a monomorphism then $\underline{f}=\underline{g}\circ\underline{h}$ is a homomorphism, i.e., the composition preserves the property of being a morphism but in general loses the injectivity of $\underline{h}$. Since $\underline{f}$ corresponds to the homomorphism the representational model assumes as the formalization of measurement, this operational-representational formalism specifies the representational model by highlighting the *internal structure* of $\underline{f}$, based on the presence of a MS, and thus integrating in a single framework both the empirical and the formal instances arising in the measurement.

4.4. Further considerations on the concept of measurement

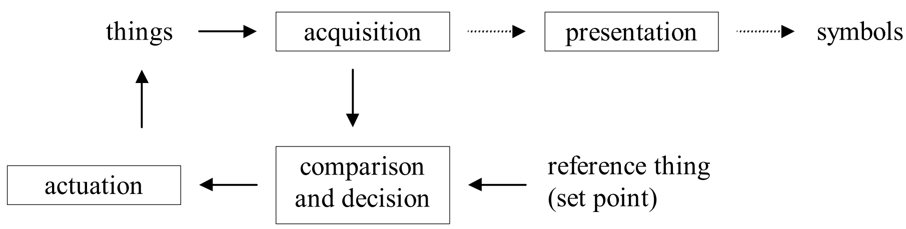
The identification of the AC-PC structure of MSs has some meaningful consequences on the very concept of measurement.

(*i*) The AC synthesizes in itself the empirical side of measurement, and as such it *determines* a relation between the thing and the chosen reference as the result of their measurand-specific comparison. Such a comparison can be performed according to one of several alternatives (the reference is embedded in the AC; the reference is external to the AC, and the AC compares the thing and the reference at the same time, or in time sequence), but in all the cases the AC output is a calibrated result, in the sense that it considers both the state of the thing and the reference. It can be noted that ACs are not peculiar to MSs, since they are present, e.g., in control systems. The proposed model makes the relations between these systems clear.

* Measuring systems:



* Control systems:



In control systems the presentation phase is optional, since the set point could be assigned in a purely empirical way, according to the strategy "keep the thing in the current state, whatever it is" (in this case the AC would produce uncalibrated results). Measurement and control have thus a different nature, and measurement is not, generally speaking, a component of control although usually adopted as such.

(*ii*) The PC synthesizes in itself the symbolic side of measurement, and as such it *assigns* a symbol representative of the AC output.

Indeed, the two requirements for $\underline{h}$ (of being injective and conserving the relations defined on Z) do not lead to its univocal definition (but in the case of absolute measurement scale, as e.g., in counting) and leave a degree of arbitrariness in the implementation of the PC. For example, if the only relation defined on Z is an order among the equivalence classes then any monotonic mapping is a monomorphism, and thus an acceptable realization of $\underline{h}$. The monomorphism $\underline{h}$ is identified but an admissible transformation for the scale type, defined as an automorphism of $\underline{\mathrm{Y}}$, i.e., an isomorphism of $\underline{\mathrm{Y}}$ into itself. If $\underline{h}:\underline{\mathrm{Z}}\rightarrow\underline{\mathrm{Y}}$ is a monomorphism and $\underline{t}:\underline{\mathrm{Y}}\rightarrow\underline{\mathrm{Y}}$ is an automorphism, then also $\underline{h}'=\underline{h}\circ\underline{t}:\underline{\mathrm{Z}}\rightarrow\underline{\mathrm{Y}}$ is a monomorphism, and therefore adequately generates a symbolic expression of the empirical information obtained by the AC. Admissible transformations thus simply realize the renaming operation corresponding to a scale change (e.g., from Celsius to Fahrenheit degrees), that has no implication on the empirical side of measurement.

Therefore, in a MS *the AC identifies the measurand and its scale type, while the PC defines the specific scale adopted to express the measurement results*.

5. Conclusions

In this paper measurement has been modeled as both, although not at the same time, a determination and an assignment. The proposed operational-representational model mediates the extreme positions of measurement as either pure determination or pure assignment, and at the same time it integrates the axiomatic-representational approach and the operational one, usually assumed as incompatible. The basis of this mediation is found in measuring systems, whose functional characteristics have been investigated and whose presence thus assumes a foundational role in the very definition of measurement: generally speaking, measurement is deemed to be the homomorphic evaluation (as claimed by the representational model) performed by means of a measuring system. The result of the empirical interaction between the thing to be measured and the measuring system is determined, and a symbol is then assigned as measurement result to formally express the empirically obtained information. Measuring systems are therefore peculiar mediators between things and symbols: this gives the measurement an intermediate status between empirical and symbolic realms, and between objectivity and subjectivity.

As a partial correction of the quoted definition by Finkelstein [5], in [1] it was proposed to consider measurement as «the process of empirical, objective assignment of symbols to things with respect to attributes, in such a way as to describe such things and their relations». We are now able to make this definition of measurement somehow more explicit, suggesting that "*measurement is the process of empirical, objective assignment of symbols to things with respect to attributes, as realized by measuring systems that perform the twofold task of empirically comparing things with given references and representing the results of such comparisons in symbolic form*".

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