A pragmatic perspective on measurement

Luca Mari
Università Cattaneo – LIUC
Italy
lmari@liuc.it

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Outline

- Introduction (to myself and the subject)
- An important problem
- Four standpoints, no (complete) solutions
- A proposal
- A related measurement model
- Concluding remarks, particularly about measurement of non-physical properties

Targeting measurement

“Those sciences, created almost in our own days, the object of which is man himself, the direct goal of which is the happiness of man, will enjoy a progress no less sure than that of the physical sciences; & this idea so sweet, that our nephews will surpass us in wisdom as in enlightenment, is no longer an illusion. In meditating on the nature of the moral sciences, one cannot help seeing that, as they are based like the physical sciences upon the observation of fact, they must follow the same method, acquire a language equally exact & precise, attaining the same degree of certainty.”

[Condorcet, 1782]

My context (1)

- TC1-Education and Training in Measurement and Instrumentation
- TC2-Photonics
- TC3-Measurement of Force, Mass and Torque
- TC4-Measurement of Electrical Quantities
- TC5-Hardness Measurement
- TC7-Measurement Science
- TC8-Traceability in Metrology
- TC9-Flow Measurement
- TC10-Technical Diagnostics
- TC11-Metrological Infrastructures
- TC12-Temperature and Thermal Measurements
- TC13-Measurements in Biology and Medicine

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- TC14-Measurement of Geometrical Quantities
- TC15-Experimental Mechanics
- TC16-Pressure and Vacuum Measurement
- TC17-Measurement in Robotics
- TC18-Measurement of Human Functions
- TC19-Environmental Measurements
- TC20-Energy Measurement
- TC21-Mathematical Tools for Measurements
- TC22-Vibration Measurement
- TC23-Metrology in Food and Nutrition
- TC24-Chemical Measurements

My context (2)

(JCGM) Joint Committee for Guides in Metrology:

- (BIPM) Int.l Bureau of Weights and Measures
- (IEC) Int.l Electrotechnical Commission
- (IFCC) Int.l Federation of Clinical Chemistry and Laboratory Medicine
- (ILAC) Int.l Laboratory Accreditation Cooperation
- (ISO) Int.l Organization for Standardization
- (IUPAC) Int.l Union of Pure and Applied Chemistry
- (IUPAP) Int.l Union of Pure and Applied Physics
- (OIML) Int.l Organization of Legal Metrology

My context (3)

the “VIM”


JCGM 200:2008

International vocabulary of metrology — Basic and general concepts and associated terms (VIM)

Vocabulaire international de métrologie — Concepts fondamentaux et généraux et termes associés (VIM)

the “GUM”


JCGM 100:2008

GUM 1995 with minor corrections

Evaluation of measurement data — Guide to the expression of uncertainty in measurement

Évaluation des données de mesure — Guide pour l'expression de l'incertitude de mesure

http://www.bipm.org/en/publications/guides/ vim.html
gum.html

My background

V.Lazzarotti, R.Manzini, LM, A model for R&D performance measurement, *Int. J. of Production Economics*, 2011

A.Frigerio, A.Giordani, LM, Outline of a general model of measurement, *Synthese*, 2010

LM, On (kinds of) quantities, *Metrologia*, 2009

P.Carbone, L.Buglione, LM, D.Petri, A comparison between foundations of Metrology and Software Measurement, *IEEE Trans. Instrumentation and Measurement*, 2008

LM, The problem of foundations of measurement, *Measurement*, 2005

LM, Epistemology of measurement, *Measurement*, 2003

LM, Beyond the representational viewpoint: a new formalization of measurement, *Measurement*, 2000

A basic hypothesis

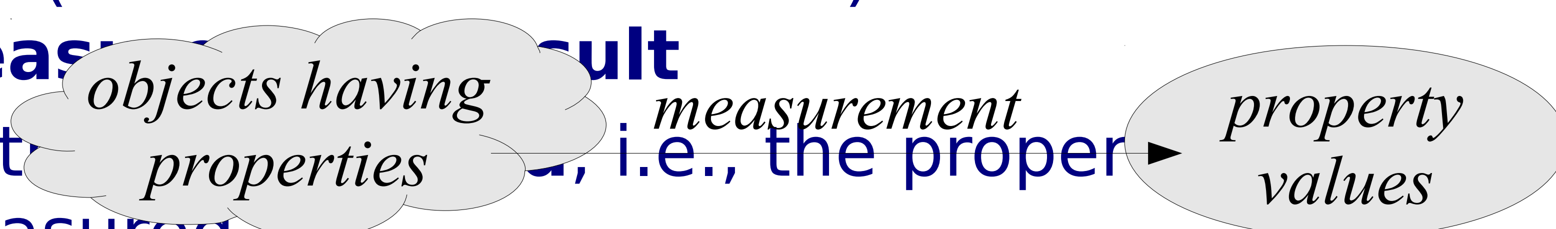
Being an infrastructural, widespread activity, performed by human beings since millennia, measurement is laden with myths

Backgrounder

How is “property” different to “measurand”? And how do these relate to the (psychometric) term “construct”?

A few basic concepts and terms:

- given an **object** (phenomenon, event
- having a **property** (attribute, observable, quantity, ...)
- **measurement** is a property-related process
- which, applied to the object, produces an **information entity**
- interpreted as a **property value**
- and (with other information) called the **meas**[illegible]**ult**
- on t[illegible]d, i.e., the proper[illegible]to be measured



Measurement is a property representation process

Backgrounder (2)

Measurement-related models typically assume that:

- there are **general properties** (*e.g., length, leadership*)
- some general properties can be considered of some objects (*leadership of a person but not of a table*)
- a general property of an object is an **individual property** of that object (*length of a given table, leadership of a given person*)
- a measurement problem is about a general property (*I would like to measure leadership*)
- measurement is applied to individual properties (*I am measuring the leadership of this person*)
- a general property is characterized by a set of property values (*positive real numbers for length*)
- an individual property is represented by a property value (the *length of this table is* 2.34 m)

Backgrounder (3

Say more about what you mean by “empirical fact”.

These assumptions lead to a functional m
involved entities where:

- general properties are described as functions,
- whose domain is a set of objects
- and whose range is a set of individual properties

$$p_{gen} : \{objects\} \rightarrow \{p_{ind}\}$$

(*your leadership* is modeled as $leadership(you)$, so that the fact that leadership is not considered of tables, i.e., $leadership(this\ table)$ is wrong, is modeled as the hypothesis that tables do not belong to the domain of the function $leadership$)

Such functions describe empirical facts

Measurement is aimed at representing individual properties

(this is written with the “=” symbol, $length(this\ table) = 2.34\ \mathrm{m}$, properly meaning “is represented by”, not “is equal to”)

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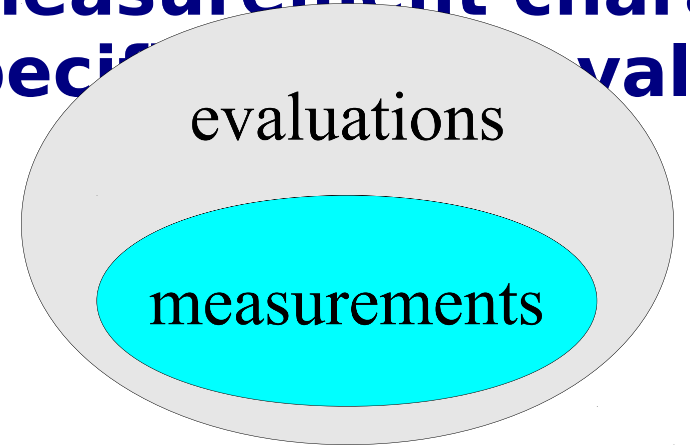
What is the problem?

The previous description applies not only to measurement,
but generically to processes of assignment of property values,
(property) evaluations for short
(i.e. it gives necessary but not sufficient conditions to define ‘measurement’)

How is measurement characterized as a speci[...] evaluation?



What is the importa

Say more about what you mean by “reliability”.

- From an **epistemic** point of view: measurement results are considered conveying “reliable” information on properties:

  what is the source of such reliability?

- From a **pragmatic** point of view: it is socially accepted that obtaining measurement results requires employing some resources:

  under what conditions is such acceptation justified?

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Is it an already solved problem?

Several standpoints...

Measurement is:

- S1: process producing Euclidean ‘measures’
- S2: physical transducer operation
- S3: morphic representation
- S4: decision making support

S1. Measurement as a process producing Euclidean ‘measures’

“A magnitude is a part of a magnitude, the less of the greater, when it measures the greater; the greater is a multiple of the less when it is measured by the less; a ratio is a sort of relation in respect of size between two magnitudes of the same kind.”

[Euclid, Elements, Book V, definitions 1-3]

→ **A property is measurable because** it can be represented by property values of the form $n \cdot u$, where n is an integer number and u is a “unit” property

S1. Significance / benefits

- This standpoint is the basis of the classical concept of **quantity** (a quantity is a property representable as multiple of a unit), and therefore of quantity calculus / dimensional analysis, where quantities are represented, in Maxwell's notation, as:

$$q = \{q\}[q]$$

q: quantity to be represented
$\{q\}$: numerical quantity value
$[q]$: unit

- The International System of Quantities (ISQ), and then the International System of Units (SI), are based on quantities in the Euclidean sense

S1. Objections

- This standpoint does not give any justification of the claimed reliability of measurement results ("according to my experience, I can see that this object is 1,2 m long" expresses in fact a ratio of two "magnitudes"; nevertheless, this is hardly acceptable as a measurement result)

- It is today customarily accepted that less-than-ratio properties (e.g., ordinal) can be measurable

→ **This characterization gives neither necessary nor sufficient conditions of measurability**

S2. Measurement as the process performed physical transduc

Give an example or two of "physical transducers".

The "geometrical paradigm" has been successfully exported to the physical world and embedded in a metrological infrastructure

→ **A property is measurable because** it is the input signal of a properly calibrated and operated instrument realizing a physical transduction effect

S2. Significance / benefits

- This standpoint emphasizes that measurability has to do with the way the information on the property is acquired, not (only) the way it is represented

➔ **This characterization could give sufficient conditions of measurability**

S2. Objections

- Although effective in measurement of physical properties, this standpoint is useless if the aim is to characterize the measurability of non-physical properties

➔ **This characterization does not give necessary conditions of measurability**

S3. Measurement as a morphic representation of properties

"Measurement is the assignment of numerals to objects or events according to rule, any rule."

[Stevens 1959]

As the outcome of a critical analysis on the possibilities of applying measurement in social sciences, measurement has been axiomatized as a morphic mapping from properties to property values (e.g., if $p(a) <_p p(b)$ then $m_p(a) < m_p(b)$)

→ **A property is measurable because** it can be mapped to a set of property values and the mapping is a morphism

S3. Significance / benefits

- This standpoint has been very fruitful in terms of its theoretical consequences, as it is the basis of the so called “representational theories of measurement”: multiple “measurement scales” are identified (e.g., nominal, ordinal, …), and for each of them a **representation** theorem (what conditions are required for a morphic mapping to be defined) and a **uniqueness** theorem (what conditions constrain the values assigned by the morphic mapping) are given

→ **This characterization gives a parametric set of (plausibly) necessary conditions of measurability**

S3. Objections

- As for the first objection to S1, this standpoint does not give any justification of the claimed reliability of measurement results (“according to my experience, I can see that the object a is shorter than b, and therefore the length value I have assigned to a is less than the length value assigned to b”; nevertheless, this is hardly acceptable as a measurement result)

- **This characterization does not give sufficient conditions of measurability**

S4. Measurement as a process supporting decision making

A measurement result is the “symbolic representation of event, state or attribute, to aid in the making of a decision”

[Nicholas, White 2001]

→ **A property is measurable because** its values, as obtained by means of measurement, are useful in decision making

S4. Significance / benefits

- This standpoint emphasizes that, as any production process, measurement should be justified in terms of the usefulness of the results it produces

→ **This characterization might give a (very loose) necessary condition of measurability**

S4. Objections

- Is it really appropriate to characterize measurement as any “useful” (?) “symbolic representation” (?)?

- **This characterization does not give actual conditions of measurability**

An open problem, then

Measurement is:

- S1: process producing Euclidean ‘measures’
- S2: physical transducer operation
- S3: morphic representation
- S4: decision making support

Several standpoints, but none of them fully appropriate to characterize a concept of measurement general enough to encompass non-physical properties but specific enough to exclude generic evaluations

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Beyond S1 – S4?

The hypothesis that measurement is a property evaluation whose results convey reliable information on the measurand is not related to:

- the nature of the object under measurement or of the measurand (*both physical and non-physical properties should be in principle measurable*)
- the algebraic structure of the set of property values (*not only Euclidean quantities should be in principle measurable*)

What is the epistemic source of such reliability which pragmatically justifies employing some resources to obtain measurement results?

Conceptual propo

This reliability is justified in terms
features expected for measureme
principle are supposed to convey i

- **specific to the measurand**, a
  any other property of the object
  environment, which includes both the measuring system and the subject who is measuring
- **interpretable in the same way by different users** in different places and times, and therefore expressed in a form independent of the specific context and only referring to entities which are universally accessible

In the phrase “specific to the measurand, and independent of any other property of the object or the surrounding environment”, would it be OK to add a few words to say the following: “specific to the measurand, and independent of the measurement of any other property of the object or the surrounding environment”.

Lexical proposa

Give some examples of objectivity and intersubjectivity.

The supposition that the information c
measurement results

- is specific to the measurand, and therefore to the object of measurement, is a requirement of **objectivity**
- is universally interpretable, and therefore is the same for different individuals, is a requirement of **intersubjectivity**

Accordingly, objectivity and intersubjectivity are independent features

(an evaluation might be objective but not intersubjective, or intersubjective but not objective)

Pragmatic propos

Perhaps, after this, review S1-S4 and mention important differences/similarities.

Neither objectivity nor intersubjectivity
measurement results are Boolean (i.e.
features

Measurement results have an overall degree of "quality", customarily expressed in terms of their (un)certainty and
related to their objectivity and intersubjectivity

Hence, if the usefulness of measurement results has to be taken into account then their uncertainty must be less than a *target measurement uncertainty* ("measurement uncertainty specified as an upper limit and decided on the basis of the intended use of measurement results"), i.e., they must be

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A simple model of measurement

Measurement:

$$p \xrightarrow{m} v$$

is a structured, empirical + representational, process, based on the possibility of **transducing** p to a property $\bar{p}$ (the “indication”):

$$p \xrightarrow{\tau_p} \bar{p}$$

and then mapping $\bar{p}$ to a value $\bar{v}$ (the “indication value”):

$$\begin{tikzcd} p \arrow[r, "\tau_p"] & \bar{p} \arrow[d] \\ & \bar{v} \end{tikzcd}$$

where the empirical mapping τ_p is required to be **calibrated**

Calibration

Let us suppose that a set of standards is available such that each of them:

- realizes a reference property r
- is associated with a given property value v_r
- is transduced to an indication $\bar{p}$, and then associated with an indication value $\bar{v}$

$$\begin{tikzcd} r \arrow[r, "\tau_p"] \arrow[d] & \bar{p} \arrow[d] \\ v_r & \bar{v} \end{tikzcd}$$

Then a mapping τ_v ("calibration function") can be construed:

$$v_r \xrightarrow{\tau_v} \bar{v}$$

Measurement (simplest, ideal version)

Under the hypothesis that:

- the calibration function can be inverted
- the transduction is stable (the function did not change)

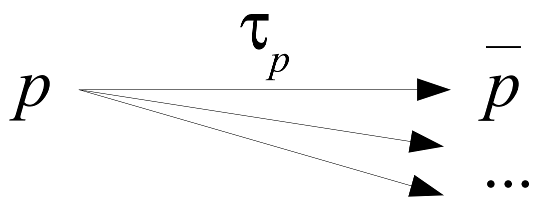
the measurement $p \rightarrow v$ is performed by:

1. transducing the measurand p to an indication $\overline{p}$
2. mapping the indication $\overline{p}$ to the indication value $\overline{v}$
3. mapping the indication value $\overline{v}$ to a measurand value v

$$
\begin{tikzcd}
p \arrow[r, "\boxed{\mathbf{1}}\ \tau_p"] \arrow[d, dashed] & \overline{p} \arrow[d, "\boxed{\mathbf{2}}"] \\
v & \overline{v} \arrow[l, "\tau_v^{-1}\ \boxed{\mathbf{3}}"']
\end{tikzcd}
$$

Measurement (more realistic version, 1)

It might be acknowledged that:



the transducer is not perfectly stable, because it is sensitive to some **influence properties** other than the measurand

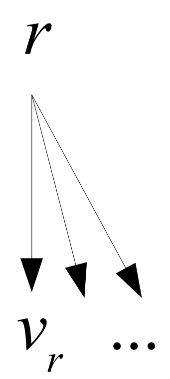
(i.e., the transducer does not behave as an ideal filter)

so that the indication $\bar{p}$ depends not only on the measurand p but also on such other properties

This reduces the objectivity of measurement: measuring systems are designed to minimize such effects and therefore to maximize objectivity

Measurement (more realistic version, 2)

It might be acknowledged that:



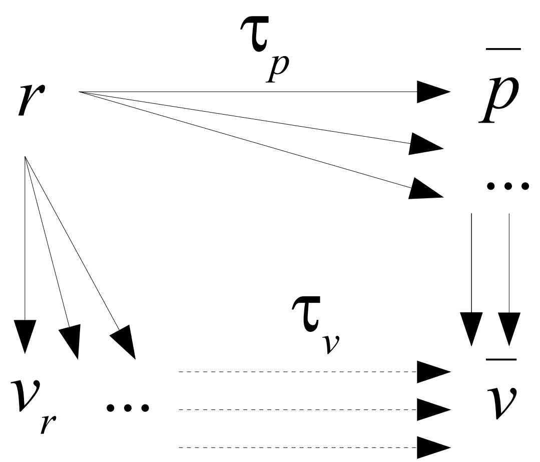
the standards are not perfectly stable, and for all non-primary standards the information on the reference property value is uncertain

so that the reference properties r are not mapped to a single property value v_r

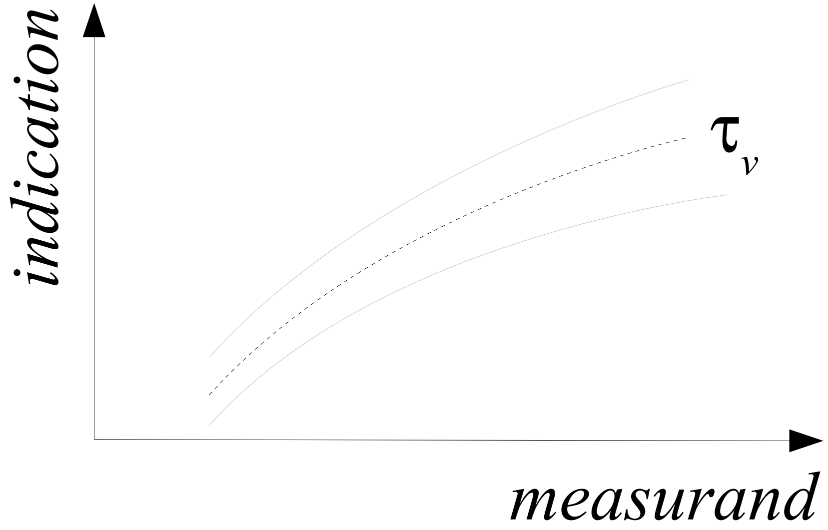
This reduces the intersubjectivity of measurement: standards are designed to minimize such effects and therefore to maximize intersubjectivity

Calibration (more realistic version)

Since:

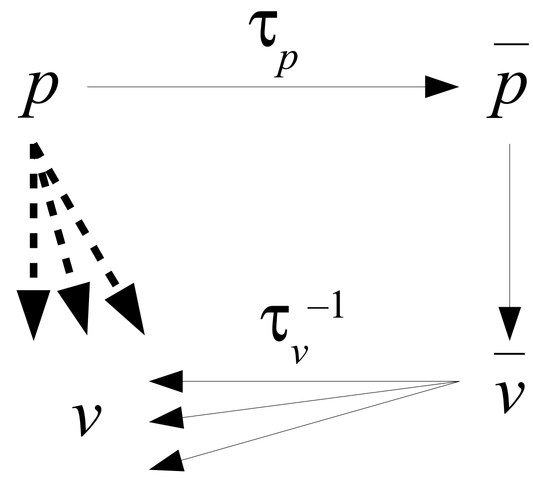


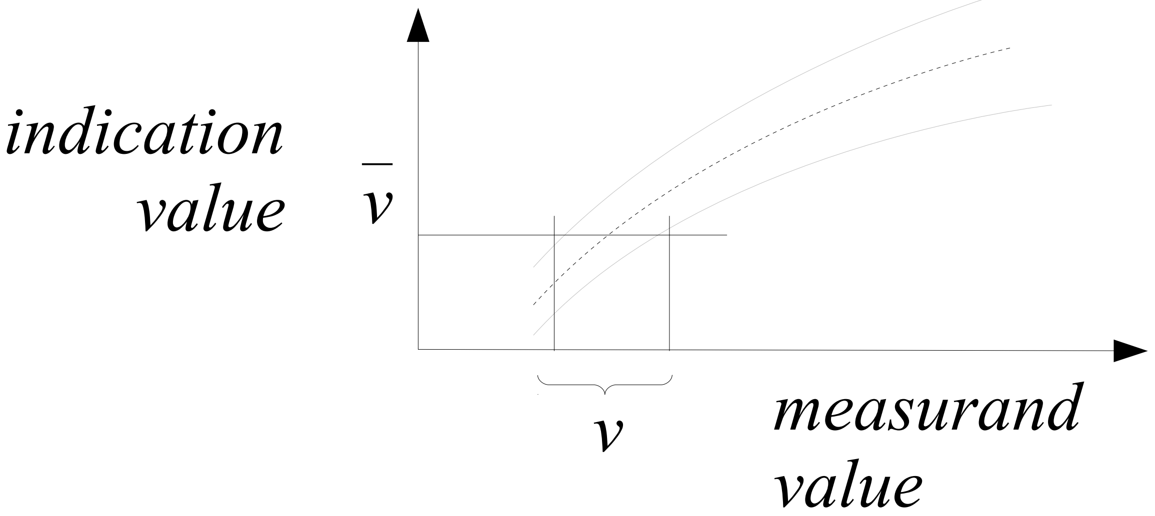
then the mapping τ_v becomes a “calibration strip”:



Measurement (more realistic version)

As a consequence, even when a single transduction is performed:





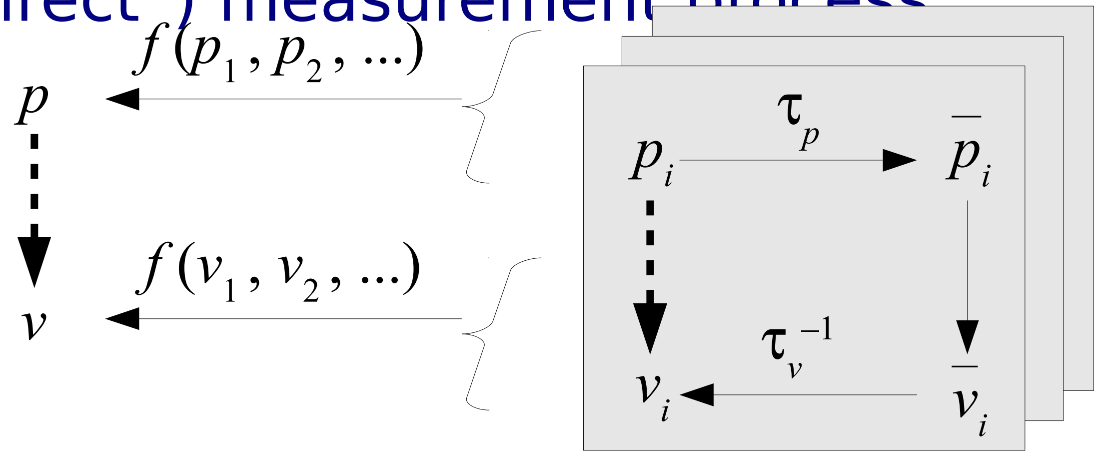
measurement results are affected by uncertainty

Pragmatically: measurement uncertainty should be less than target uncertainty

Measurement (extended version)

In a more general case, the measurand p might be not the input property of a transducer, but is dependent, through a given function f, on one or more properties p_i that can be transduced (or whose values are somehow known)

The previous process becomes a component of the whole ("indirect") measurement process:



Hence, the uncertainty on the values v_i must be "propagated" through f to compute the uncertainty of v

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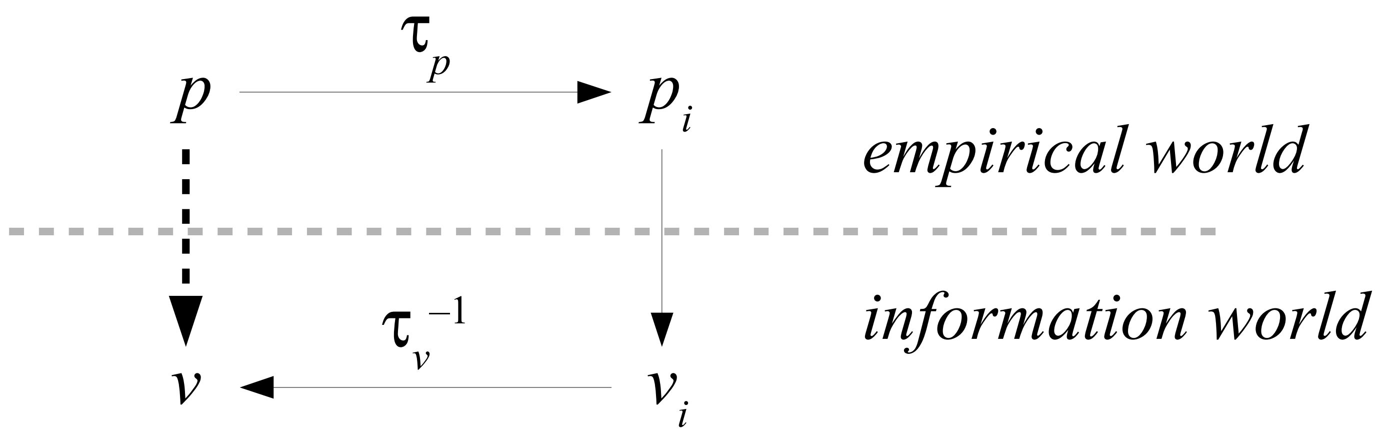
The problem of measurand

Measurement bridges two worlds...



The last step is to **interpret** the obtained property value, v, as a value of the measurand, i.e., the property **intended** to be measured:

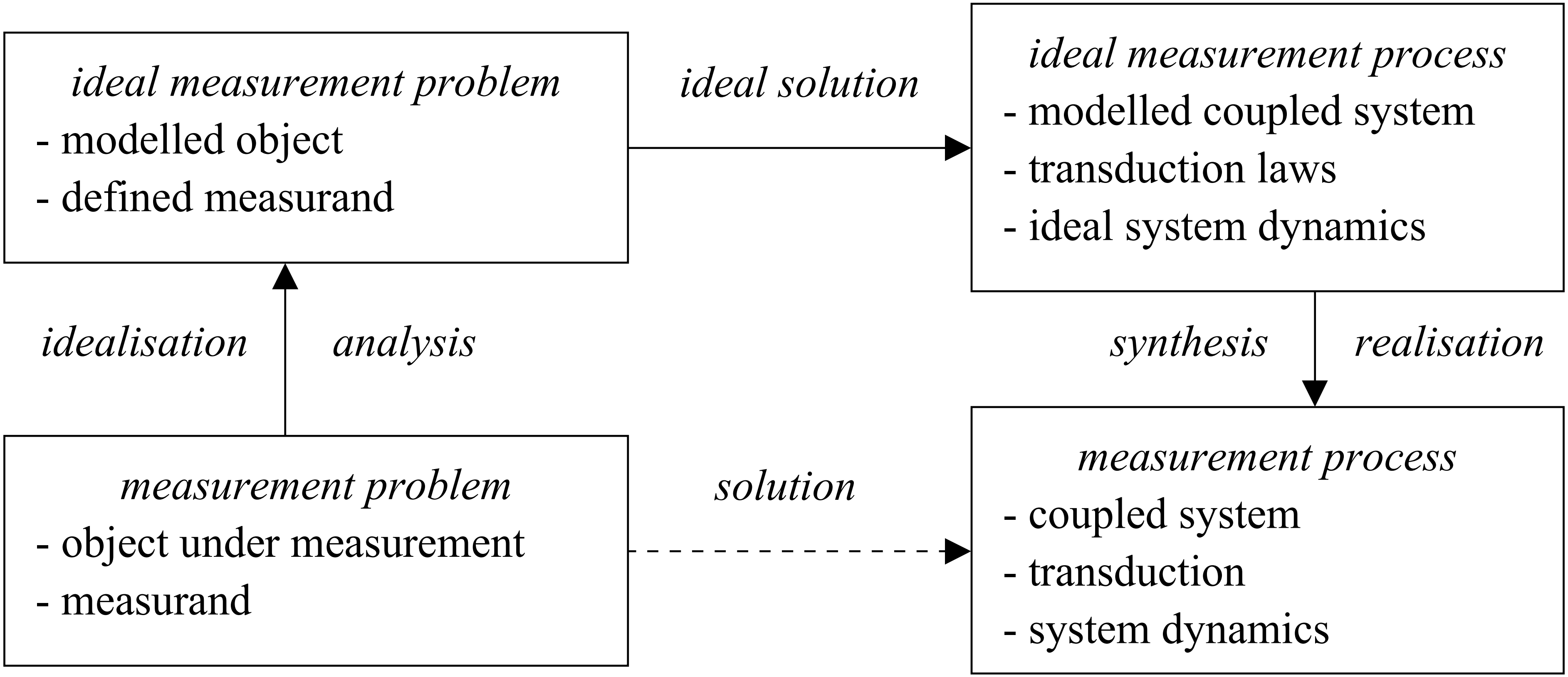
is the property which I have measured
actually the property which I intended to measure?

‘Definitional uncertainty’

“component of measurement uncertainty resulting from the finite amount of detail in the definition of a measurand”

Note: “definitional uncertainty is the practical minimum measurement uncertainty achievable in any measurement of a given measurand”

Models



A source of uncertainty comes from the “correspondence” between the problem/process and its model

Measurement of non-physical properties

(just to trigger the discussion)

Nothing in this presentation implies the physical nature of measurands; hence this analysis and its conclusions seem to be applicable also to non-physical properties

Nevertheless, the some differences (typically) remain

Differences...

Slide 49: What would be a likely alternative to “descriptive”?

1. Physical quantities are mutually rela
laws; this allows:

- minimizing primitive (“purely operational”) concepts
- cross-validating measurand definitions
- cross-checking measurements results

2. A global metrological infrastructure is well established for physical quantities

3. The measurement of physical properties is a purely descriptive process

4. Physical properties have been measured since millennia

A pragmatic perspective on measurement

Thank you for the kind attention

Luca Mari

lmari@liuc.it