Workshops

- Wednesday, November 20, 12:00 PM to 2:00 PM: Models of measurement: the general structure
- Thursday, November 21, 9:00 AM to 11:00 AM: Models of measurement: measuring systems and metrological infrastructure
- Thursday, November 21, 2:00 PM to 4:00 PM: An overview on measurement uncertainty: from the standpoint of the Guide to the Expression of Uncertainty in Measurement (GUM)
- Friday, November 22, 10:00 AM to noon: Is the body of knowledge on measurement worth to be a 'science', and what may be the scope of a measurement science?

Workshop 2 Models of measurement: measuring systems and metrological infrastructure

> Luca Mari Università Cattaneo – LIUC, Italy

> University of California, Berkeley Thursday, November 21, 2013

Abstract

Building upon the proposed epistemological characterization, the workshop focuses on the structural features of measuring systems, front-ends of a metrological infrastructure and tools designed and operated so to guarantee a required minimum level of objectivity and intersubjectivity for the conveyed information. This highlights the twofold nature of measurement, an information acquisition and representation process in which the role of models is unavoidable, even though possibly left implicit in the simplest cases.

My profile

Luca Mari (M.Sc. in physics; Ph.D. in measurement science) is full professor of measurement science at the Cattaneo University – LIUC, Castellanza (VA), Italy, where he teaches courses on measurement science, statistical data analysis, system theory.

He is currently the chairman of the TC1 (Terminology) and the secretary of the TC25 (Quantities and Units) of the International Electrotechnical Commission (IEC), and an IEC expert in the WG2 (VIM) of the Joint Committee for Guides in Metrology (JCGM). He has been the chairman of the TC7 (Measurement Science) of the International Measurement Confederation (IMEKO). He is the author or coauthor of several scientific papers published in international journals and international conference proceedings. His research interests include measurement science and system theory.

Some of my recent publications

LM, A quest for the definition of measurement, Measurement, 2013

- LM, A.Giordani, Quantity and quantity value, Metrologia, 2012
- LM, P.Carbone, D.Petri, Measurement fundamentals: a pragmatic view, IEEE Trans. Instr. Meas., 2012
- A.Giordani, LM, Measurement, models, uncertainty, IEEE Trans. Instr. Meas., 2012
- A.Giordani, LM, Property evaluation types, Measurement, 2012
- A.Frigerio, A.Giordani, LM, Outline of a general model of measurement, Synthese, 2010
- D.Macii, LM, D.Petri, Comparison of measured quantity value estimators in nonlinear models, *IEEE Trans. Instr. Meas.*, 2010
- LM, V.Lazzarotti, R.Manzini, Measurement in soft systems: epistemological framework and a case study, *Measurement*, 2009
- LM, A computational system for uncertainty propagation of measurement results, *Measurement*, 2009
- LM, On (kinds of) quantities, Metrologia, 2009
- LM, The problem of foundations of measurement, Measurement, 2005
- LM, Epistemology of measurement, Measurement, 2003
- LM, Beyond the representational viewpoint: a new formalization of measurement, *Measurement*, 2000

Basic entities

Our ontology involves four kinds of entities:

 $\Omega = \{\omega_i\}$: objects *P* : general properties (objects in Ω are *P*-comparable) *P*(ω_i) : individual properties (*P* of ω_i) *V* = {*v*_i} : property values

where the set of standard objects $S = \{s_i\} \subset \Omega$ is such that:

- if $i \neq j$ then $P(s_i) \neq P(s_i)$
- $\forall \omega_i \in \Omega, \exists ! s_i \in S \text{ such that either } P(\omega_i) \approx P(s_i)$
- the s_i are P-stable and easily P-clonable to easily accessible objects, so that the equivalence classes [P(s_i)] are worth to be identified as v_i being P(s_i) (individual) reference properties

Basic relations

Our ontology involves three kinds of relations:

- between individual properties: P(ω_i) ≈ P(ω_j) an experimental comparison
- between property values: v_i ≈ v_j
 a formal equality
- between an individual property and a property value: P(ω) = v a v-assignment



«The theory of measurement is difficult enough without bringing in the theory of making measurements» [H. Kyburg, Theory and measurement, 1984]

«We are not interested in a measuring apparatus and in the interaction between the apparatus and the objects being measured. Rather, we attempt to describe how to put measurement on a firm, well-defined foundation» [F. Roberts, Measurement theory, 1979] «... a certain deficiency of realism in philosophical discussions of measurement»

[O.D. Duncan, Notes on social measurement. Historical and critical, 1984]

«If you wish to elucidate theoretical concepts, such as that of quantity, look at the way they function in theories. But if you wish to clarify pragmatic concepts, such as that of measurement, look also at practice.»

> [M. Bunge, On confusing 'Measure' with 'Measurement' in the methodology of behavioral science, 1973]

That is why I will avoid the term "measurement theory"... Conditions for measurement

"Direct" (synchronous) measurement

"Indirect" (asynchronous) measurement

Hints for extending the model

Notes on transducers as measuring instruments

From v-assignment to measurement

Measurement is an informative property v-assignment:

- a value assignment
- to a property
- so to convey information on it

But not each informative property v-assignment is a measurement (e.g., subjective judgment and guess can also be informative property vassignments, but usually they are not expected to be measurements):

how is measurement characterized with respect to a generic v-assignment?

Conditions for measurement

An option space...



For a conceptual history of measurement...



Exploring the option D

Measurement as an **informative property v-assignment** that delivers information:

- specifically related to the measurand and not to some other properties of the object under measurement or the empirical environment, which includes also the subject who is measuring → it is a condition object-relatedness, i.e., of objectivity
- univocally interpretable by different users in different places and times, thus implying that a measurement result has to be unambiguous and unambiguously expressed
 → it is a condition of subject-transparency,
 - i.e., of intersubjectivity

Measurement and measuring systems

When measuring a physical property, these conditions are guaranteed by the measurement system itself:

- the output of the measuring instrument ideally depends only on the property under measurement, and it is independent of all other properties of the empirical environment → this confers objectivity to the provided information
- the measuring instrument is calibrated against a measurement standard, thus making measurement results traceable so that different measuring instruments calibrated within the same metrological system provide compatible information
 → this confers intersubjectivity to the provided information

A tentative definition

Measurement is a both conceptual and experimental process

implementing a v-assignment

able to produce information on a predefined property

with a specified and provable level of objectivity and intersubjectivity

Let us work on this

(at first under the supposition that uncertainties can be ignored)

Conditions for measurement

"Direct" (synchronous) measurement

"Indirect" (asynchronous) measurement

Hints for extending the model

Notes on transducers as measuring instruments

From comparison to value assignment

1. The object under consideration, ω , is *P*-compared with the standard objects in S

2. For a given k, the standard s_k is identified such that $P(\omega) \approx P(s_k)$

3. The corresponding value v_k is reported: $P(\omega) = v_k$ in V



... an example of what in algebra is called a commutative diagram

"Direct" measurement

Let us suppose that the *P*-comparison is performed by an instrument:



object under measurement *a* having the property *P*(ω) (the **measurand**) measurement standard *s* realizing the property *P*(*s*) (a **reference property**)

It is the structural strategy customarily known as **direct** (or better: **synchronous**) **measurement** because based on a synchronous comparison



P-comparison as P-selection

The process of *P*-comparison:



can be functionally intended as the selection of a reference property:



... so that the structure of the whole process becomes:



Measuring instruments as filters

Generally the result of the *P*-selection depends not only on

- the property subject to measurement $P(\omega)$
- and the reference properties $P(s_i)$

but also on other properties:

- $Q_1(\omega)$, of the object under measurement
- $Q_2(s_i)$, of the standard objects
- $Q_3(.)$, of the measuring instrument
- $Q_4(.)$, of the environment

globally called influence properties



On the other hand, the result of the *P*-selection is expected to be related to $P(\omega)$ and $\{P(s_i)\}$ only:

a basic task for a measuring instrument is to operate as a filter, preventing the effects of influence properties to be propagated to the measurement result

Measuring instruments as partial filters

Generally the effects of some influence properties P_x cannot be avoided on the result of *P*-selection

At least three structural strategies can be alternatively adopted: • report the result with respect to given reference conditions $P_{x, ref}(.)$ by measuring the current conditions $P_x(.)$ and applying a "correction model" if $P_x(.) \neq P_{x, ref}(.)$

report the result with respect to the current conditions P_x(.)
 after having measured them

• report the result **in terms of "average" conditions** possibly after the repetition of the *P*-selection

Measuring instruments and information objectivity

The inappropriate treatment of influence properties limits the object-relatedness of the information provided by the measuring instrument

Given the structure of synchronous measurement by *P*-comparison / *P*-selection...



... it remains to explore the step (2)

Mapping reference properties to values

In principle, the only constraint in the mapping is injectivity, aimed at preventing information loss: since we are able to *P*-distinguish the standard objects in *S*, to each $P(s_i)$ a different v_i should be associated



Standard set^(*) construction

Let us consider again the conditions on the set *S* of standard objects: 1. if $i \neq j$ then $P(s_i) \neq P(s_j)$

2. $\forall \omega_i \in \Omega, \exists ! s_j \in S$ such that either $P(\omega_i) \approx P(s_j)$

3. the objects s_i are easily *P*-clonable to easily accessible objects, so that the equivalence classes [$P(s_i)$] are worth to be identified as v_i

1 (mutual exclusivity) and 2 (exhaustivity) specify that S defines a partition on the set of properties $P(\omega_i)$,

and 3 that the partition is easily replicable

... it is like throwing a net on a plane of points s_i according to the criterion *P*:



S

 $[P(s_i)]$ to which v_i is assigned

(*) Someone would call it "scale", a terribly polysemous term...

More on standard set construction

In fact, in "typical" cases *P* is (known to be) such that an additive operation \oplus is defined so that triples of objects *x*, *y*, *z* can be generally found such that $(P(x) \oplus P(y)) \approx P(z)$

In these cases S can be built as:

- 1. selection of a "unit" standard, s_1
- 2. cloning of s_1
- 3. construction of s_2 such that $P(s_2) \approx (P(s_1) \oplus P(s_1))$
- ... iterative construction of s_i such that $P(s_i) \approx (P(s_{i-1}) \oplus P(s_1))$

The assignment of reference values can be done accordingly:

- $S_1 \rightarrow V_1$
- $s_2 \rightarrow 2V_1$
- $\dots s_n \rightarrow n v_1$

The distinction between "nominal" and "additive" v-assignment relates to standard set construction

Stability

A critical condition for measurement is that its results are interpreted in the same way in different times and places

This requires the reference value v_i to be stably associated to the standard object s_i such that $P(s_k) = v_k$ so that everytime everywhere $P(\omega) = v_k$ means $P(\omega) \approx P(s_k)$



Reach

For "universal" measurement stability is then necessary but not sufficient: standard objects must be *P*-clonable...



... so that from $P(\omega_1) = v_k$ and $P(\omega_2) = v_k$ the conclusion $P(\omega_1) \approx P(\omega_2)$ can be drawn even if ω_1 and ω_2 have never been *P*-compared directly

Standard objects dissemination



Each cloning process is a standard calibration

This is the basic structure of the metrological system

Traceability chains and traceability

metrological traceability chain: «sequence of measurement standards and calibrations that is used to relate a measurement result to a reference»

metrological traceability: «property of a measurement result whereby the result can be related to a reference through a documented unbroken chain of calibrations, each contributing to the measurement uncertainty»

[VIM3]



Measuring instruments and information intersubjectivity

The inappropriate calibration of working standards limits the subject-transparency of the information provided by the measuring instrument

Measurement as an inferential process

IF the primary standard s_k is identified and $P(s_k) = v_k$ AND the working standard s^{**}_k is constructed as a *P*-clone of s_k , $P(s_k) \approx P(s^{**}_k)$ AND the object *a* is *P*-compared with a set of working standards so that $P(\omega) \approx P(s^{**}_k)$ THEN the measurement result is $P(\omega) = v_k$ Posterior v-assignment

Even this simple model must assume that measurement results are the product of an inference

Remarks

In this inferential process:

nothing requires quantification / additivity:

in principle the process operates identically in the ordinal and even in the nominal case

 nothing implies the experimental comparison to be performed by a physical device or relatively to physical properties:

> in principle, the procedure operates identically for physical and non-physical properties

Conditions for measurement

"Direct" (synchronous) measurement

"Indirect" (asynchronous) measurement

Hints for extending the model

Notes on transducers as measuring instruments

"Direct" measurement: shortcomings



This procedure requires the availability of:

- measurement standards when measurement is performed ("direct" comparison actually means synchronous comparison)
- references of the same order of magnitude of the measurand
- a device able to compare quantities of that order of magnitude

P-transduction

These shortcomings are overcome when measurement is performed according to a different, and actually much more widespread, procedure

The property subject to measurement is applied to a device (called "transducer", or more specifically "sensor") which produces another property in response (typically by changing its state):



It is assumed that such output property R(.)(traditionally called "**instrument reading**" or "**indication**") conveys sufficient information on $P(\omega)$



Transducers, indications

indicating measuring instrument: «measuring instrument providing an output signal carrying information about the value of the quantity being measured»

indication: «quantity value provided by a measuring instrument or a measuring system»

[VIM3]

(I will assume instead that indications are properties, not property values)

P-transduction of tool of measurement

The transducer is a device built, setup, and operated on purpose so to implement / realize a **transduction effect**

The transducer is designed so to transform the measurement problem on *P* into a (supposedly simpler) measurement problem on *R*

But this is still not sufficient to prevent a never-ending recursion...

"Primitively solvable" measurement problems

Let us suppose that the measurement problem on R is "primitively solvable", i.e., the v-assignment R(.) = w is somehow given ... typically because obtained by:

 direct / synchronous measurement or:

counting of easily identifiable entities



But the value assigned to $P(\omega)$ must be a *P*-value, not a *R*-value...

From transduction to value assignment

The structure of this kind of measurement is then:



1

The core concept is the **divide and conquer strategy**:

- convert 4 to another measurement problem 2 by means of
- solve 2
- use the solution to 2 to solve 4 by means of 3

Principles, methods, procedures

measurement principle: «phenomenon serving as a basis of a measurement»

measurement method: «generic description of a logical organization of operations used in a measurement»

measurement procedure: «detailed description of a measurement according to one or more measurement principles and to a given measurement method, based on a measurement model and including any calculation to obtain a measurement result»

[VIM3]

Transduction model

The transduction is an experimental process involving empirical properties, modeled by an informational process involving mathematical entities: let us assume it is a function f from P-values to R-values



Under the supposition that f is invertible,

 f^{-1} is the sought mapping 3 from *R*-values to *P*-values

Modeling transduction behavior

The behavior of τ can be modeled by:

- a defined non-parametric function *f* (white box model)
- a defined parametric function *f*, whose parameter values are not known (gray box model)
 → how to obtain information on the parameters of *f* ?
- a generic, undefined function *f* (black box model)
 → how to obtain information on *f* ?

Instrument calibration

A set S of calibrated standard objects s, is available

(i.e., the v-assignment $P(s_i) = v_i$ is given) and:

- $P(s_i)$ is transduced to R(.) and then mapped to a R-value w_i
- the pairs $\langle v_i, w_i \rangle$ are used "to reconstruct" (by fitting, interpolation, ...) f



f is so important that deserves multiple (!) names: **transduction function, observation function, calibration function**

Standard calibration and instrument calibration

Hence, two distinct concepts of calibration are relevant to measurement:

- calibration of standard objects, aimed at assigning a property value to each object of the standard set
- calibration of transducer, aimed at defining the calibration function for the transducer by means of calibrated standard objects

(is some sort of calibration required for comparators too?)

Calibration and measurement

Instrument calibration and measurement are inverse processes:



"Indirect" measurement

This measurement procedure still «implies comparison of quantities», even though in an asynchronous way:



This shows that a basic metrological requirement for a transducer is its **stability**: «property of a measuring instrument, whereby its metrological properties remain constant in time» [VIM3]

Measurement as an inferential process

IF the primary standard s_k is identified and $P(s_k) = v_k$ AND the working standard s^{**}_k is constructed as a *P*-clone of s_k , $P(s_k) \approx P(s^{**}_k)$ AND the transducer is calibrated so that w_k is the indication that corresponds to v_k AND the property $P(\omega)$ is transduced and the obtained indication value is w_k

THEN the measurement result is $P(\omega) = v_{k}$

prior v-assignment & standard calibration & instrument calibration & transduction ↓ posterior v-assignment

Even this (relatively) simple model must assume that measurement results are the product of an inference

Remarks

In this inferential process:

nothing requires quantification / additivity:

in principle the process operates identically in the ordinal and even in the nominal case

 nothing implies the transduction to be performed by a physical device or relatively to physical properties:

in principle, the procedure operates identically for physical and non-physical properties

Conditions for measurement

"Direct" (synchronous) measurement

"Indirect" (asynchronous) measurement

Hints for extending the model

Notes on transducers as measuring instruments

Extending the model

This model of a transduction as the experimental component of measurement is a simplified one, for at least three reasons:

A) it assumes the transducer to be a perfect filter with respect to the measurand

B) it assumes the measurand to be the input quantity to the transducerC) it does not take into account possible loading effects on the measurand

(i.e., the model assumes that the transducer output conveys information on the unaltered measurand (C), on the whole measurand (B), only on the measurand (A))

Let us consider how the model could be made more realistic

A) Transducers as filters

The model $P(.) \rightarrow R(.)$ is a simplified one because it assumes that the transducer interacts with $P(\omega)$ only:

$$P(\omega) \longrightarrow \tau \longrightarrow R(.)$$

But this is generally not the case, due to the effects of influence properties:

so that the functional behavior of the device is actually twofold, a transduction τ preceded by a filtering φ :

that in the ideal case operates so to remove the dependence of R on any property other than P

A) Transducers as non-ideal filters

The fact that no ideal filter are available implies that in principle R(.) depends on both $P(\omega)$ and some influence properties (**influence quantity**: «quantity that, in a direct measurement, does not affect the quantity that is actually measured, but affects the relation between the indication and the measurement result» [VIM3])

When this further, undesired dependence is recognized as critical, at least two strategies are available:

- the influence properties are measured in their turn, and the measurement result is reported by also specifying them; on the other hand this dependence is specific of the given transducer and therefore (i) not necessarily so significant in a general context, and however (ii) significant only if the transducer had been calibrated in the same environmental conditions
- the mapping v → w keeps into account also such influence properties (typically dealt with as parameters), so that it can be inverted in the same parametric (i.e., for the same environmental) conditions

B) Measurands as input quantities

The model $P(\omega) \rightarrow R(.)$ is a simplification, because it assumes that the input property to the transducer is actually the measurand In a more general case, the measurand $P(\omega)$ might be not the input property of a transducer, but is dependent, through a function g, on one or more "(partial) indicator" properties $P_j(\omega)$ that can be transduced (or whose values are somehow known)

The whole measurement process becomes then:

$$P = g(P_1, P_2, ...)$$



where the measurand is here properly "a construct", characterized by the function g and possibly accepted as "non-observable"

C) Loading effects of transducers

Ideally the interaction between the object under measurement ω and the transducer τ is unidirectional:

$$P(\omega) \longrightarrow \tau \longrightarrow R(.)$$

i.e., the state of ω is not affected by the interaction with τ

But this is not generally the case, the interaction being instead:

$$P(\omega) \longrightarrow R(.)$$
loading

so that what is measured is not the unaffected state of ω but a modified version of it (despite the operative similarity, this has nothing to do with the situations "of signal conditioning" in which ω has to be modified on purpose before measurement so to make the measurand properly accessible to τ) When the loading effect is known on the measurand, a "correction" can be introduced in the inverted version of $v \rightarrow w$ so to keep into account of it

C) The "Hawthorne effect"

In physical systems the loading effects can produce random or systematic modifications on the state of *a*, but in either case it is maintained that *a* does not behave according to any optimization principle / criterion

In social systems a specific kind of loading effect is the so-called "Hawthorne effect", where ω is acknowledged to be an intentional entity, so that the state of *a* might change according to an optimality criterion (e.g., to maximize the measurand, or to minimize its difference to a given "target" value in order to fulfill some explicit or implicit expectation)

This effect may become particularly significant if the measurement is repeated, so that *a* can "adapt" to the measurement conditions

It might be exploited as a management tool to drive *a* towards a given state, with the seemingly paradoxical consequence that a measurement system could be designed, setup, and operated not to acquire information on the object ω but to have the state of a "self-changed" because of this feedback

Conditions for measurement

"Direct" (synchronous) measurement

"Indirect" (asynchronous) measurement

Hints for extending the model

Notes on transducers as measuring instruments

Synchronic and diachronic transitivity

The condition of transitivity of (synchronous or asynchronous) *P*-comparison \approx :

 $(P(s_k) \approx P(s^{**}_{k})) \land (P(\omega) \approx P(s^{**}_{k})) \rightarrow (P(\omega) \approx P(s_k))$ so that $P(s_k) = v_k \rightarrow P(\omega) = v_k$

is delicate:

• in synchronic terms:

 $(x \approx y) \land (y \approx z) \rightarrow (x \approx z)$ remember, e.g., the well-known problems related to the sorites paradox

in diachronic terms:

 $(x(t_1) \approx x(t_2)) \land (x(t_2) \approx x(t_3)) \rightarrow (x(t_1) \approx x(t_3))$ because it implies perfect stability

(and nevertheless without transitivity (traditional) mathematics cannot be applied)

Invertibility of the calibration function

The calibration function f is used in its inverted form f^{-1} in measurement: but is f invertible?

The condition is more easily understood in the case $\{P(.)\}$ is discrete

The transduction $P(.) \rightarrow R(.)$ must maintain the differences: if $P(x) \neq P(y)$ then also $R(x) \neq R(y)$ a condition (of injectivity) of **information preservation**

More generally, this is related to instrument **resolution**: «smallest change in a quantity being measured that causes a perceptible change in the corresponding indication» [VIM3]

Transduction of a casual relation

Even though in measurement the transduction $P(.) \rightarrow R(.)$ is eventually aimed at being inverted, the inversion is only required in its informational counterpart, $v \rightarrow w$, and indeed the transduction is generally non-invertible

This asymmetry, together with the assumption that indications are produced together with, or after, the interaction object under measurement – transducer (but not before it), gives a default interpretation of the **transduction as a cause-effect relation**

Discovering transduction effects

While the transducer is a device built and setup on purpose, the transduction effect is assumed as empirically given

This arises the problem: how are transduction effects discovered?

This is interesting also because through a transduction effect two or more quantities are functionally connected with each other, so that transduction effects might be considered as the gluing elements of a nomological network

Transducers and "observability"

Why "indirect" (asynchronous) measurement a so more widespread strategy than "direct" (synchronous) one?

We are interested in performing measurements in much more, and much more diverse, situations that in the past, and relatively to measurands that are outside the range or resolution of human senses or to which humans senses are not sensitive at all, a case that traditionally would have been described as of "non-directly observable" quantities

In an epistemological perspective the primacy of unaided human senses is questionable (visually observing by means of glasses makes the observation "less direct"?)

If instead by "direct observability" of a quantity is meant the possibility of empirically interacting with it, then transducers can be thought of as means to extend the number of "directly observable" quantities

Remarks, again

Measurement is an inferential process such that:

• nothing requires quantification / additivity in it:

in principle the process operates identically in the ordinal and even in the nominal case

 nothing implies its experimental component to be performed by a physical device or relatively to physical properties:

> in principle, the procedure operates identically for physical and non-physical properties

this paves the way to develop social and physical measurement science on a common foundation...

THANK YOU FOR YOUR KIND ATTENTION

Luca Mari Imari@liuc.it